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Synchronous Generators: Modeling for (and) Transients

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5.1 Introduction

The previous chapter dealt with the principles of synchronous generators (SGs) and steady state based on the two-reaction theory. In essence, the concept of traveling field (rotor) and stator magnetomotive forces (mmfs) and airgap fields at standstill with each other has been used.

By decomposing each stator phase current under steady state into two components, one in phase with the electromagnetic field (emf) and the other phase shifted by 90° , two stator mmfs, both traveling at rotor speed, were identified. One produces an airgap field with its maximum aligned to the rotor poles (d axis), while the other is aligned to the q axis (between poles).

The d and q axes magnetization inductances X_{dm} and X_{qm} are thus defined. The voltage equations with balanced three-phase stator currents under steady state are then obtained.

Further on, this equation will be exploited to derive all performance aspects for steady state when no currents are induced into the rotor damper winding, and the field-winding current is direct. Though unbalanced load steady state was also investigated, the negative sequence impedance $\underline{Z}_-$ could not be explained theoretically; thus, a basic experiment to measure it was described in the previous chapter. Further on, during transients, when the stator current amplitude and frequency, rotor damper and field currents, and speed vary, a more general (advanced) model is required to handle the machine behavior properly.

Advanced models for transients include the following:

- Phase-variable model
- Orthogonal-axis (d - q) model
- Finite-element (FE)/circuit model

The first two are essentially lumped circuit models, while the third is a coupled, field (distributed parameter) and circuit, model. Also, the first two are analytical models, while the third is a numerical model. The presence of a solid iron rotor core, damper windings, and distributed field coils on the rotor of nonsalient rotor pole SGs (turbogenerators, $2p_1 = 2, 4$), further complicates the FE/circuit model to account for the eddy currents in the solid iron rotor, so influenced by the local magnetic saturation level.

In view of such a complex problem, in this chapter, we are going to start with the phase coordinate model with inductances (some of them) that are dependent on rotor position, that is, on time. To get rid of rotor position dependence on self and mutual (stator/rotor) inductances, the d - q model is used. Its derivation is straightforward through the Park matrix transform. The d - q model is then exploited to describe the steady state. Further on, the operational parameters are presented and used to portray electromagnetic (constant speed) transients, such as the three-phase sudden short-circuit.

An extended discussion on magnetic saturation inclusion into the d - q model is then housed and illustrated for steady state and transients.

The electromechanical transients (speed varies also) are presented for both small perturbations (through linearization) and for large perturbations, respectively. For the latter case, numerical solutions of state-space equations are required and illustrated.

Mechanical (or slow) transients such as SG free or forced "oscillations" are presented for electromagnetic steady state.

Simplified d - q models, adequate for power system stability studies, are introduced and justified in some detail. Illustrative examples are worked out. The asynchronous running is also presented, as it is the regime that evidentiates the asynchronous (damping) torque that is so critical to SG stability and control. Though the operational parameters with $s = \omega j$ lead to various SG parameters and time constants, their analytical expressions are given in the design chapter (Chapter 7), and their measurement is presented as part of Chapter 8, on testing.

This chapter ends with some FE/coupled circuit models related to SG steady state and transients.

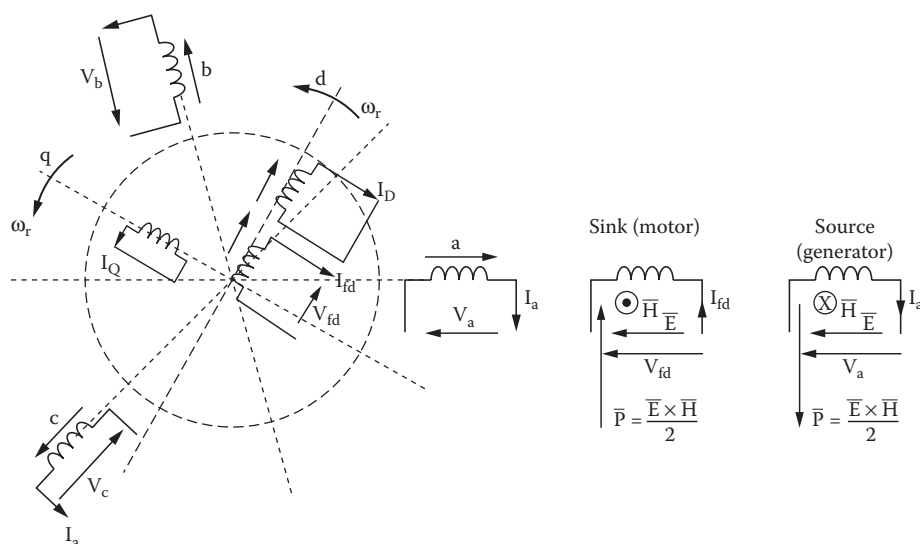


FIGURE 5.1 Phase-variable circuit model with single damper cage.

5.2 The Phase-Variable Model

The phase-variable model is a circuit model. Consequently, the SG is described by a set of three stator circuits coupled through motion with two (or a multiple of two) orthogonally placed (d and q) damper windings and a field winding (along axis d : of largest magnetic permeance; see Figure 5.1). The stator and rotor circuits are magnetically coupled with each other. It should be noticed that the convention of voltage–current signs (directions) is based on the respective circuit nature: source on the stator and sink on the rotor. This is in agreement with Poynting vector direction, toward the circuit for the sink and outward for the source (Figure 5.1).

The phase-voltage equations, in stator coordinates for the stator, and rotor coordinates for the rotor, are simply missing any “apparent” motion-induced voltages:

$$\begin{aligned} i_A R_s + v_a &= -\frac{d\Psi_A}{dt} \\ i_B R_s + v_b &= -\frac{d\Psi_B}{dt} \\ i_C R_s + v_c &= -\frac{d\Psi_c}{dt} \\ i_D R_D &= -\frac{d\Psi_D}{dt} \\ i_Q R_Q &= -\frac{d\Psi_Q}{dt} \\ I_f R_f - V_f &= -\frac{d\Psi_f}{dt} \end{aligned} \quad (5.1)$$

The rotor quantities are not yet reduced to the stator. The essential parts missing in Equation 5.1 are the flux linkage and current relationships, that is, self- and mutual inductances between the six coupled circuits in Figure. 5.1. For example,

In a similar way,

$$L_{BB} = L_{sl} + L_0 + L_2 \cos\left(2\theta_{er} + \frac{2\pi}{3}\right) \quad (5.8)$$

$$L_{CC} = L_{sl} + L_0 + L_2 \cos\left(2\theta_{er} - \frac{2\pi}{3}\right) \quad (5.9)$$

The mutual inductance between phases is considered to be only in relation to airgap permeances. It is evident that, with ideally (sinusoidally) distributed windings, $L_{AB}(\theta_{er})$ varies with θ_{er} as L_{CC} and again has two components (to a first approximation):

$$L_{AB} = L_{BA} = L_{AB0} + L_{AB2} \cos\left(2\theta_{er} - \frac{2\pi}{3}\right) \quad (5.10)$$

Now, as phases A and B are 120° phase shifted, it follows that

$$L_{AB0} \approx L_0 \cos \frac{2\pi}{3} = -\frac{L_0}{2} \quad (5.11)$$

The variable part of L_{AB} is similar to that of Equation 5.9 and thus,

$$L_{AB2} = L_2 \quad (5.12)$$

Relationships 5.11 and 5.12 are valid for ideal conditions. In reality, there are some small differences, even for symmetric windings. Further,

$$L_{AC} = L_{CA} = -\frac{L_0}{2} + L_2 \cos\left(2\theta_{er} + \frac{2\pi}{3}\right) \quad (5.13)$$

$$L_{BC} = L_{CB} = -\frac{L_0}{2} + L_2 \cos 2\theta_{er} \quad (5.14)$$

FE analysis of field distribution with only one phase supplied with direct current (DC) could provide ground for more exact approximations of self- and mutual stator inductance dependence on θ_{er} . Based on this, additional terms in $\cos(4\theta_{er})$, even $6\theta_{er}$, may be added. For fractionary q windings, more intricate θ_{er} dependences may be developed.

The mutual inductances between stator phases and rotor circuits are straightforward, as they vary with $\cos(\theta_{er})$ and $\sin(\theta_{er})$.

$$\begin{aligned} L_{Af} &= M_f \cos \theta_{er} \\ L_{Bf} &= M_f \cos\left(\theta_{er} - \frac{2\pi}{3}\right) \\ L_{Cf} &= M_f \cos\left(\theta_{er} + \frac{2\pi}{3}\right) \end{aligned} \quad (5.15)$$

$$\begin{aligned}
L_{AD} &= M_D \cos \theta_{er} \\
L_{BD} &= M_D \cos \left(\theta_{er} - \frac{2\pi}{3} \right) \\
L_{CD} &= M_D \cos \left(\theta_{er} + \frac{2\pi}{3} \right) \\
L_{AQ} &= -M_Q \sin \theta_{er} \\
L_{BQ} &= -M_Q \sin \left(\theta_{er} - \frac{2\pi}{3} \right) \\
L_{CQ} &= -M_Q \sin \left(\theta_{er} + \frac{2\pi}{3} \right)
\end{aligned} \tag{5.15 cont.}$$

Notice that

$$\begin{aligned}
L_0 &= \frac{(L_{dm} + L_{qm})}{2} \\
L_2 &= \frac{(L_{dm} - L_{qm})}{2}
\end{aligned} \tag{5.16}$$

L_{dm} and L_{qm} were defined in Chapter 4 with all stator phases on, and M_f is the maximum of field/armature inductance also derived in Chapter 4.

We may now define the SG phase-variable 6×6 matrix $\left[L_{ABCfDQ}(\theta_{er}) \right]$:

	A	B	C	f	D	Q
A	$L_{sl} + L_0 + L_2 \cos 2\theta_{er}$	$-\frac{L_0}{2} + L_2 \cdot \cos \left(2\theta_{er} - \frac{2\pi}{3} \right)$	$-\frac{L_0}{2} + L_2 \cdot \cos \left(2\theta_{er} + \frac{2\pi}{3} \right)$	$M_f \cos \theta_{er}$	$M_D \cos \theta_{er}$	$-M_Q \cos \theta_{er}$
B	$L_{sl} + L_0 + L_2 \cos \left(2\theta_{er} + \frac{2\pi}{3} \right)$	$-\frac{L_0}{2} + L_2 \cdot \cos 2\theta_{er}$	$M_f \cos \left(\theta_{er} - \frac{2\pi}{3} \right)$	$M_D \cos \left(\theta_{er} - \frac{2\pi}{3} \right)$	$-M_Q \sin \left(\theta_{er} - \frac{2\pi}{3} \right)$	
C	$L_{sl} + L_0 + L_2 \cos \left(2\theta_{er} - \frac{2\pi}{3} \right)$	$M_f \cos \left(\theta_{er} + \frac{2\pi}{3} \right)$	$M_D \cos \left(\theta_{er} + \frac{2\pi}{3} \right)$	$-M_Q \sin \left(\theta_{er} - \frac{2\pi}{3} \right)$		
f				$L'_f + L'_{fm}$	M'_{fD}	0
D					$L'_{Dl} + L'_{Dm}$	0
Q				0	0	$L'_{Ql} + L'_{Qm}$

(5.17)

A mutual coupling leakage inductance L_{fDl} also occurs between the field winding f and the d -axis cage winding D in salient-pole rotors. The zeroes in Equation 5.17 reflect the zero coupling between orthogonal windings in the absence of magnetic saturation. L'_{fm} , L'_{Dm} , L'_{Qm} are typical main (airgap permeance) self-inductances of rotor circuits. L'_f , L'_{Dl} , L'_{Ql} are the leakage inductances of rotor circuits in axes d and q .

The resistance matrix is of diagonal type:

$$R_{ABCfdq} = \text{Diag} \left[R_s, R_r, R_s, R_f, R'_D, R'_Q \right] \quad (5.18)$$

Provided core losses, space harmonics, magnetic saturation, and frequency (skin) effects in the rotor core and damper cage are all neglected, the voltage/current matrix equation fully represents the SG at constant speed:

$$\begin{aligned} \left[I_{ABCfDQ} \right] \left[R_{ABCfDQ} \right] + \left[V_{ABCfDQ} \right] &= \frac{-d\Psi_{ABCfDQ}}{dt} = \\ &- \left[L_{ABCfDQ}(\theta_{er}) \right] \frac{d}{dt} \left[I_{ABCfDQ} \right] - \frac{\partial \left[L_{ABCfDQ} \right]}{\partial \theta_{er}} \frac{d\theta_{er}}{dt} \left[I_{ABCfDQ} \right] \end{aligned} \quad (5.19)$$

with

$$V_{ABCfDQ} = \left[+V_A, +V_B, +V_C, -V_f, 0, 0 \right]^T; \frac{d\theta_{er}}{dt} = \omega_r \quad (5.20)$$

$$\Psi_{ABCfDQ} = \left[\Psi_A, \Psi_B, \Psi_C, \Psi_f^r, \Psi_D^r, \Psi_Q^r \right]^T \quad (5.21)$$

The minus sign for V_f arises from the motor association of signs convention for rotor.

The first term on the right side of Equation 5.19 represents the transformer-induced voltages, and the second term refers to the motion-induced voltages.

Multiplying Equation 5.19 by $[I_{ABCfDQ}]^T$ yields the following:

$$\begin{aligned} \left[I_{ABCfDQ} \right]^T \left[V_{ABCfDQ} \right] &= -\frac{1}{2} \left[I_{ABCfDQ} \right]^T \frac{\partial \left[L_{ABCfDQ}(\theta_{er}) \right]}{\partial \theta_{er}} \left[I_{ABCfDQ} \right] \cdot \omega_r - \\ &- \frac{d}{dt} \left[\frac{1}{2} \left[I_{ABCfDQ} \right]^T \cdot L_{ABCfDQ}(\theta_{er}) \cdot \left[I_{ABCfDQ} \right] \right] - \left[I_{ABCfDQ} \right]^T \left[I_{ABCfDQ} \right] \left[R_{ABCfDQ} \right] \end{aligned} \quad (5.22)$$

The instantaneous power balance equation (Equation 5.22) serves to identify the electromagnetic power that is related to the motion-induced voltages:

$$P_{elm} = -\frac{1}{2} \left[I_{ABCfDQ} \right]^T \cdot \frac{\partial}{\partial \theta_{er}} \left[L_{ABCfDQ}(\theta_{er}) \right] \left[I_{ABCfDQ} \right] \omega_r \quad (5.23)$$

P_{elm} should be positive for the generator regime.

The electromagnetic torque T_e opposes motion when positive (generator model) and is as follows:

$$T_e = \frac{+P_e}{(\omega_r / p_1)} = -\frac{p_1}{2} \left[I_{ABCfDQ} \right]^T \frac{\partial \left[L_{ABCfDQ}(\theta_{er}) \right]}{\partial \theta_{er}} \left[I_{ABCfDQ} \right] \quad (5.24)$$

The equation of motion is

$$\frac{J}{p_1} \frac{d\omega_r}{dt} = T_{shaft} - T_e; \frac{d\theta_{er}}{dt} = \omega_r \quad (5.25)$$

The phase-variable equations constitute an eighth-order model with time-variable coefficients (inductances). Such a system may be solved as it is either with flux linkages vector as the variable or with the current vector as the variable, together with speed ω_r and rotor position θ_{er} as motion variables.

Numerical methods such as Runge–Kutta–Gill or predictor-corrector may be used to solve the system for various transient or steady-state regimes, once the initial values of all variables are given. Also, the time variations of voltages and of shaft torque have to be known. Inverting the matrix of time-dependent inductances at every time integration step is, however, a tedious job. Moreover, as it is, the phase-variable model offers little in terms of interpreting the various phenomena and operation modes in an intuitive manner.

This is how the d – q model was born — out of the necessity to quickly solve various transient operation modes of SGs connected to the power grid (or in parallel).

5.3 The d – q Model

The main aim of the d – q model is to eliminate the dependence of inductances on rotor position. To do so, the system of coordinates should be attached to the machine part that has magnetic saliency — the rotor for SGs.

The d – q model should express both stator and rotor equations in rotor coordinates, aligned to rotor d and q axes because, at least in the absence of magnetic saturation, there is no coupling between the two axes. The rotor windings f , D , Q are already aligned along d and q axes. The rotor circuit voltage equations were written in rotor coordinates in Equation 5.1.

It is only the stator voltages, V_A , V_B , V_C , currents I_A , I_B , I_C , and flux linkages Ψ_A , Ψ_B , Ψ_C that have to be transformed to rotor orthogonal coordinates. The transformation of coordinates ABC to d – q 0, known also as the Park transform, valid for voltages, currents, and flux linkages as well, is as follows:

$$\begin{bmatrix} P(\theta_{er}) \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos(-\theta_{er}) & \cos\left(-\theta_{er} + \frac{2\pi}{3}\right) & \cos\left(-\theta_{er} - \frac{2\pi}{3}\right) \\ \sin(-\theta_{er}) & \sin\left(-\theta_{er} + \frac{2\pi}{3}\right) & \sin\left(-\theta_{er} - \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad (5.26)$$

So,

$$\begin{bmatrix} V_d \\ V_q \\ V_0 \end{bmatrix} = \begin{bmatrix} P(\theta_{er}) \end{bmatrix} \cdot \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} \quad (5.27)$$

$$\begin{bmatrix} I_d \\ I_q \\ I_0 \end{bmatrix} = \begin{bmatrix} P(\theta_{er}) \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \quad (5.28)$$

$$\begin{bmatrix} \Psi_d \\ \Psi_q \\ \Psi_0 \end{bmatrix} = \begin{bmatrix} P(\theta_{er}) \end{bmatrix} \cdot \begin{bmatrix} \Psi_A \\ \Psi_B \\ \Psi_C \end{bmatrix} \quad (5.29)$$

The inverse transformation that conserves power is

$$\left[P(\boldsymbol{\theta}_{er}) \right]^{-1} = \frac{3}{2} \left[P(\boldsymbol{\theta}_{er}) \right]^T \quad (5.30)$$

The expressions of Ψ_A , Ψ_B , Ψ_C from the flux/current matrix are as follows:

$$\left| \Psi_{ABC/DQ} \right| = \left| L_{ABC/DQ}(\boldsymbol{\theta}_{er}) \right| \left| I_{ABC/DQ} \right| \quad (5.31)$$

The phase currents I_A , I_B , I_C are recovered from I_d , I_q , I_0 by

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \frac{3}{2} \left[P(\boldsymbol{\theta}_{er}) \right]^T \cdot \begin{bmatrix} I_d \\ I_q \\ I_0 \end{bmatrix} \quad (5.32)$$

An alternative Park transform uses $\sqrt{\frac{2}{3}}$ instead of $2/3$ for direct and inverse transform. This one is fully orthogonal (power direct conservation).

The rather short and elegant expressions of Ψ_d , Ψ_q , Ψ_0 are obtained as follows:

$$\begin{aligned} \Psi_d &= \left(L_{sl} + L_0 - L_{AB0} + \frac{3}{2} L_2 \right) I_d + M_f I_f^r + M_D I_D^r \\ \Psi_q &= \left(L_{sl} + L_0 - L_{AB0} - \frac{3}{2} L_2 \right) I_q + M_Q I_q^r \\ \Psi_0 &= (L_{sl} + L_0 + 2L_{AB0}) I_0; L_{AB0} \approx -L_0 / 2 \end{aligned} \quad (5.33)$$

From Equation 5.16,

$$\begin{aligned} L_{dm} &= \frac{3}{2} (L_0 + L_2); \\ L_{qm} &= \frac{3}{2} (L_0 - L_2) \end{aligned} \quad (5.34)$$

are exactly the “cyclic” magnetization inductances along axes d and q as defined in [Chapter 4](#). So, Equation 5.33 becomes

$$\begin{aligned} \Psi_d &= L_d I_d + M_f I_f^r + M_D I_D^r; \\ L_d &= L_{sl} + L_{dm} \end{aligned} \quad (5.35)$$

$$\begin{aligned} \Psi_q &= L_q I_q + M_Q I_Q^r; \\ L_q &= L_{sl} + L_{qm} \end{aligned} \quad (5.36)$$

$$\Psi_0 \approx L_{sl} I_0 \quad (5.37)$$

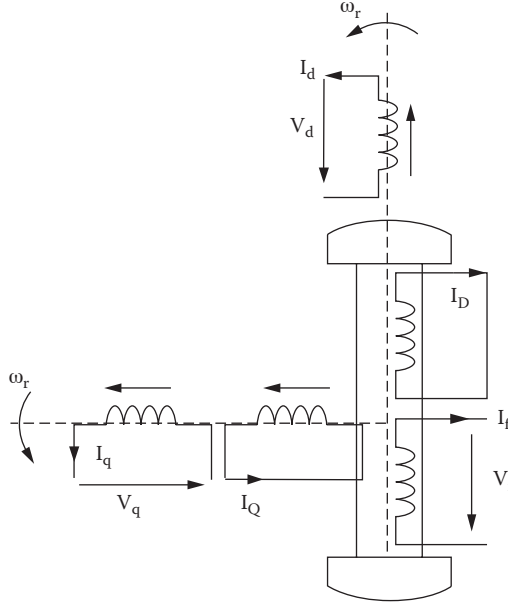


FIGURE 5.3 The d - q model of synchronous generators.

In a similar way for the rotor,

$$\begin{aligned}\Psi_f^r &= (L_{fl}^r + L_{fm}^r) I_f^r + \frac{3}{2} M_f I_d + M_{fD} I_D^r \\ \Psi_D^r &= (L_{Dl}^r + L_{Dm}^r) I_D^r + \frac{3}{2} M_D I_d + M_{fD} I_f^r \\ \Psi_Q^r &= (L_{Ql}^r + L_{Qm}^r) I_Q^r + \frac{3}{2} M_Q I_q\end{aligned}\quad (5.38)$$

As seen in Equation 5.37, the zero components of stator flux and current Ψ_0, I_0 are related simply by the stator phase leakage inductance L_{sl} ; thus, they do not participate in the energy conversion through the fundamental components of mmfs and fields in the SGs.

Thus, it is acceptable to consider it separately. Consequently, the d - q transformation may be visualized as representing a fictitious SG with orthogonal stator axes fixed magnetically to the rotor d - q axes. The magnetic field axes of the respective stator windings are fixed to the rotor d - q axes, but their conductors (coils) are at standstill (Figure 5.3) — fixed to the stator. The d - q model equations may be derived directly through the equivalent fictitious orthogonal axis machine (Figure 5.3):

$$\begin{aligned}I_d R_s + V_d &= -\frac{d\Psi_d}{dt} + \omega_r \Psi_q \\ I_q R_s + V_q &= -\frac{d\Psi_q}{dt} - \omega_r \Psi_d\end{aligned}\quad (5.39)$$

The rotor equations are then added:

$$\begin{aligned}
I_f R_f - V_f &= -\frac{d\Psi_f}{dt} \\
i_D R_D &= -\frac{d\Psi_D}{dt} \\
i_Q R_Q &= -\frac{d\Psi_Q}{dt}
\end{aligned} \tag{5.40}$$

In Equation 5.39, we assumed that

$$\begin{aligned}
\frac{d\Psi_d}{d\theta_{er}} &= -\Psi_q \\
\frac{d\Psi_q}{d\theta_{er}} &= \Psi_d
\end{aligned} \tag{5.41}$$

The assumptions are true if the windings d - q are sinusoidally distributed and the airgap is constant but with a radial flux barrier along axis d . Such a hypothesis is valid for distributed stator windings to a good approximation if only the fundamental airgap flux density is considered. The null (zero) component equation is simply as follows:

$$\begin{aligned}
I_0 R_s + V_0 &= -L_{sl} \frac{di_0}{dt} = -\frac{d\Psi_0}{dt}; \\
I_0 &= \frac{(I_A + I_B + I_C)}{3}
\end{aligned} \tag{5.42}$$

The equivalence between the real three-phase SG and its d - q model in terms of instantaneous power, losses, and torque is marked by the 2/3 coefficient in Park's transformation:

$$\begin{aligned}
V_A I_A + V_B I_B + V_C I_C &= \frac{3}{2} (V_d I_d + V_q I_q + 2V_0 I_0) \\
T_e &= -\frac{3}{2} p_1 (\Psi_d I_q - \Psi_q I_d)
\end{aligned} \tag{5.43}$$

$$R_s (I_A^2 + I_B^2 + I_C^2) = \frac{3}{2} R_s (I_d^2 + I_q^2 + 2I_0^2) \tag{5.44}$$

The electromagnetic torque, T_e , calculated in Equation 5.43, is considered positive when opposite to motion. Note that for the Park transform with $\sqrt{\frac{2}{3}}$ coefficients, the power, torque, and loss equivalence in Equation 5.43 and Equation 5.44 lack the 3/2 factor. Also, in this case, Equation 5.38 has $\sqrt{\frac{3}{2}}$ instead of 3/2 coefficients.

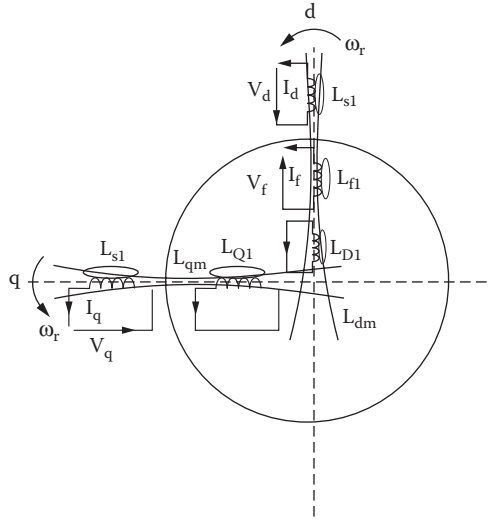


FIGURE 5.4 Inductances of d - q model.

The motion equation is as follows:

$$\frac{J}{p_1} \frac{d\omega_r}{dt} = T_{shaft} + \frac{3}{2} p_1 (\Psi_d I_q - \Psi_q I_d) \quad (5.45)$$

Reducing the rotor variables to stator variables is common in order to reduce the number of inductances. But first, the d - q model flux/current relations derived directly from Figure 5.4, with rotor variables reduced to stator, would be

$$\begin{aligned} \Psi_d &= L_{sl} I_d + L_{dm} (I_d + I_D + I_f) \\ \Psi_q &= L_{sl} I_q + L_{qm} (I_q + I_Q) \\ \Psi_f &= L_{fl} I_f + L_{dm} (I_d + I_D + I_f) \\ \Psi_D &= L_{Dl} I_D + L_{dm} (I_d + I_D + I_f) \\ \Psi_Q &= L_{Ql} I_Q + L_{qm} (I_q + I_Q) \end{aligned} \quad (5.46)$$

The mutual and self-inductances of airgap (main) flux linkage are identical to L_{dm} and L_{qm} after rotor to stator reduction. Comparing Equation 5.38 with Equation 5.46, the following definitions of current reduction coefficients are valid:

$$\begin{aligned} I_f &= I_f^r \cdot K_f \\ I_D &= I_D^r \cdot K_D \\ I_Q &= I_Q^r \cdot K_Q \\ K_f &= \frac{M_f}{L_{dm}} \end{aligned} \quad (5.47)$$

$$\begin{aligned}
 K_D &= \frac{M_D}{L_{dm}} \\
 K_Q &= \frac{M_Q}{L_{Qm}}
 \end{aligned}
 \tag{5.47 cont.}$$

We may now use coefficients in Equation 5.38 to obtain the following:

$$\Psi_f^r \cdot \frac{2}{3} \frac{L_{dm}}{M_f} = \Psi_f = L_{f\ell} I_f + L_{dm} (I_f + I_D + I_d) \tag{5.48}$$

with

$$\begin{aligned}
 L_{f\ell} &= \frac{2}{3} L_{f\ell}^r \cdot \frac{L_{dm}^2}{M_f^2} = L_{f\ell}^r \frac{2}{3 K_f^2} \\
 L_{fm} \frac{2}{3} \frac{L_{dm}}{M_f} &\approx 1 \\
 \frac{2}{3} \frac{L_{dm}}{M_f M_D} &\approx 1
 \end{aligned}
 \tag{5.49}$$

$$\Psi_D^r \cdot \frac{2}{3} \frac{L_{dm}}{M_D} = \Psi_D = L_{D\ell} I_D + L_{dm} (I_f + I_D + I_d) \tag{5.50}$$

with

$$\begin{aligned}
 L_{D\ell} &= \frac{2}{3} L_{D\ell}^r \frac{L_{dm}^2}{M_D^2} = L_{D\ell}^r \cdot \frac{2}{3} \cdot \frac{1}{K_D^2} \\
 L_{Dm} \cdot \frac{2}{3} \frac{L_{dm}}{M_D^2} &\approx 1
 \end{aligned}
 \tag{5.51}$$

$$\Psi_Q^r \frac{2}{3} \frac{L_{qm}}{M_Q} = \Psi_Q = L_{Q\ell} I_Q + L_{qm} (I_q + I_Q) \tag{5.52}$$

with

$$\begin{aligned}
 L_{Q\ell} &= \frac{2}{3} L_{Q\ell}^r \cdot \left(\frac{L_{qm}}{M_Q} \right)^2 = L_{Q\ell}^r \frac{2}{3} \frac{1}{K_Q^2} \\
 L_{Qm} \cdot \frac{2}{3} \frac{L_{qm}}{M_Q^2} &\approx 1
 \end{aligned}
 \tag{5.53}$$

We still need to reduce the rotor circuit resistances R_f^r, R_D^r, R_Q^r and the field-winding voltage to stator quantities. This may be done by power equivalence as follows:

$$\begin{aligned}\frac{3}{2}R_f(I_f^2) &= R_f^r I_f^{r2} \\ \frac{3}{2}R_D(I_D^2) &= R_D^r I_D^{r2}\end{aligned}\quad (5.54)$$

$$\begin{aligned}\frac{3}{2}R_Q(I_Q^2) &= R_Q^r I_Q^{r2} \\ \frac{3}{2}V_f I_f &= V_f^r I_f^r\end{aligned}\quad (5.55)$$

Finally,

$$\begin{aligned}R_f &= R_f^r \frac{2}{3} \frac{1}{K_f^2} \\ R_D &= R_D^r \frac{2}{3} \frac{1}{K_D^2} \\ R_Q &= R_Q^r \frac{2}{3} \frac{1}{K_Q^2} \\ V_f &= V_f^r \frac{2}{3} \frac{1}{K_f}\end{aligned}\quad (5.56)$$

Notice that resistances and leakage inductances are reduced by the same coefficients, as expected for power balance.

A few remarks are in order:

- The “physical” d - q model in [Figure 5.4](#) presupposes that there is a single common (main) flux linkage along each of the two orthogonal axes that embraces all windings along those axes.
- The flux/current relationships (Equation 5.46) for the rotor make use of stator-reduced rotor current, inductances, and flux linkage variables. In order to be valid, the following approximations have to be accepted:

$$\begin{aligned}L_{fm} L_{dm} &\approx \frac{3}{2} M_f^2 \\ M_{fD} L_{dm} &\approx \frac{3}{2} M_f M_D \\ L_{Dm} L_{dm} &\approx \frac{3}{2} M_D^2 \\ L_{Qm} L_{qm} &\approx \frac{3}{2} M_Q^2\end{aligned}\quad (5.57)$$

- The validity of the approximations in Equation 5.57 is related to the condition that airgap field distribution produced by stator and rotor currents, respectively, is the same. As far as the space fundamental is concerned, this condition holds. Once heavy local magnetic saturation conditions occur (Equation 5.57), there is a departure from reality.

- No leakage flux coupling between the d axis damper cage and the field winding ($L_{fdl} = 0$) was considered so far, though in salient-pole rotors, $L_{fdl} \neq 0$ may be needed to properly assess the SG transients, especially in the field winding.
- The coefficients K_p, K_D, K_Q used in the reduction of rotor voltage (V_f^r), currents I_f^r, I_D^r, I_Q^r , leakage inductances $L_{fl}^r, L_{Dl}^r, L_{Ql}^r$, and resistances R_f^r, R_D^r, R_Q^r , to the stator may be calculated through analytical or numerical (field distribution) methods, and they may also be measured. Care must be exercised, as K_p, K_D, K_Q depend slightly on the saturation level in the machine.
- The reduced number of inductances in Equation 5.46 should be instrumental in their estimation (through experiments).

Note that when $\sqrt{\frac{2}{3}}$ is used in the Park transform (matrix), K_p, K_D, K_Q in Equation 5.47 all have to be

multiplied by $\sqrt{\frac{3}{2}}$, but the factor $2/3$ (or $3/2$) disappears completely from Equation 5.48 through Equation 5.57 (see also Reference [1]).

5.4 The per Unit (P.U.) d - q Model

Once the rotor variables ($V_f^r, I_f^r, I_D^r, I_Q^r, R_f^r, R_D^r, R_Q^r, L_{fl}^r, L_{Dl}^r, L_{Ql}^r$) have been reduced to the stator, according to relationships 5.47, 5.54, 5.55, and 5.56, the P.U. d - q model requires base quantities only for the stator.

Though the selection of base quantities leaves room for choice, the following set is widely accepted:

$$V_b = V_n \sqrt{2} \quad \text{— peak stator phase nominal voltage} \quad (5.58a)$$

$$I_b = I_n \sqrt{2} \quad \text{— peak stator phase nominal current} \quad (5.58b)$$

$$S_b = 3V_n I_n \quad \text{— nominal apparent power} \quad (5.59)$$

$$\omega_b = \omega_m \quad \text{— rated electrical angular speed } (\omega_m = p_1 \Omega_m) \quad (5.60)$$

Based on this restricted set, additional base variables are derived:

$$T_{eb} = \frac{S_b \cdot p_1}{\omega_b} \quad \text{— base torque} \quad (5.61)$$

$$\Psi_b = \frac{V_b}{\omega_b} \quad \text{— base flux linkage} \quad (5.62)$$

$$Z_b = \frac{V_b}{I_b} = \frac{V_n}{I_n} \quad \text{— base impedance (valid also for resistances and reactances)} \quad (5.63)$$

$$L_b = \frac{Z_b}{\omega_b} \quad \text{— base inductance} \quad (5.64)$$

Inductances and reactances are the same in P.U. values. Though in some instances time is also provided with a base quantity $t_b = 1/\omega_b$, we chose here to leave time in seconds, as it seems more intuitive.

The inertia is, consequently,

$$H_b = \frac{1}{2} J \left(\frac{\omega_b}{P_1} \right)^2 \cdot \frac{1}{S_b} \quad (5.65)$$

It follows that the time derivative in P.U. terms becomes

$$\frac{d}{dt} \rightarrow \frac{1}{\omega_b} \frac{d}{dt}; s \rightarrow \frac{s}{\omega_b} \quad (\text{Laplace operator}) \quad (5.66)$$

The P.U. variables and coefficients (inductances, reactances, and resistances) are generally denoted by lowercase letters.

Consequently, the P.U. d - q model equations, extracted from Equation 5.39 through Equation 5.41, Equation 5.43, and Equation 5.46, become

$$\begin{aligned} \frac{1}{\omega_b} \frac{d}{dt} \Psi_d &= \omega_r \Psi_q - i_d r_s - v_d; \Psi_d = l_{sl} i_d + l_{dm} (i_d + i_D + i_f) \\ \frac{1}{\omega_b} \frac{d}{dt} \Psi_q &= -\omega_r \Psi_d - i_q r_s - v_q; \Psi_q = l_{sl} i_d + l_{qm} (i_q + i_Q) \\ \frac{1}{\omega_b} \frac{d}{dt} \Psi_0 &= -i_0 r_0 - v_0 \\ \frac{1}{\omega_b} \frac{d}{dt} \Psi_f &= -i_f r_f + v_f; \Psi_f = l_{fl} i_f + l_{dm} (i_Q + i_D + i_F) \\ \frac{1}{\omega_b} \frac{d}{dt} \Psi_D &= -i_D r_D; \Psi_D = l_{Dl} i_D + l_{dm} (i_d + i_D + i_F) \\ \frac{1}{\omega_b} \frac{d}{dt} \Psi_Q &= -i_Q r_Q; \Psi_Q = l_{Ql} i_Q + l_{qm} (i_q + i_Q) \\ 2H \frac{d}{dt} \omega_r &= t_{shaft} - t_e; t_{shaft} = \frac{T_{shaft}}{T_{eb}}; t_e = \frac{T_e}{T_{eb}} \\ t_e &= -(\Psi_d i_q - \Psi_q i_d); \frac{1}{\omega_b} \frac{d\theta_{er}}{dt} = \omega_r; \theta_{er} - \text{in radians} \end{aligned} \quad (5.67)$$

with t_e equal to the P.U. torque, which is positive when opposite to the direction of motion (generator mode).

The Park transformation (matrix) in P.U. variables basically retains its original form. Its usage is essential in making the transition between the real machine and d - q model voltages (in general). $v_d(t)$, $v_q(t)$, $v_f(t)$, and $t_{shaft}(t)$ are needed to investigate any transient or steady-state regime of the machine. Finally, the stator currents of the d - q model (i_d, i_q) are transformed back into i_A, i_B, i_C so as to find the real machine stator currents behavior for the case in point.

The field-winding current I_f and the damper cage currents I_D, I_Q are the same for the d - q model and the original machine. Notice that all the quantities in Equation 5.67 are reduced to stator and are, thus, directly related in P.U. quantities to stator base quantities.

In Equation 5.67, all quantities but time t and H are in P.U. measurements. (Time t and inertia H are given in seconds, and ω_b is given in rad/sec.) Equation 5.67 represents the d - q model of a three-phase

SG with single damper circuits along rotor orthogonal axes d and q . Also, the coupling of windings along axes d and q , respectively, is taking place only through the main (airgap) flux linkage.

Magnetic saturation is not yet included, and only the fundamental of airgap flux distribution is considered.

Instead of P.U. inductances $l_{dm}, l_{qm}, l_d, l_q, l_{Dl}, l_{Ql}$, the corresponding reactances may be used: $x_{dm}, x_{qm}, x_{fd}, x_{Dl}, x_{Ql}$, as the two sets are identical (in numbers, in P.U.). Also, $l_d = l_{sl} + l_{dm}$, $x_d = x_{sl} + x_{dm}$, $l_q = l_{sl} + l_{dm}$, $x_q = x_{sl} + x_{qm}$.

5.5 The Steady State via the d - q Model

During steady state, the stator voltages and currents are sinusoidal, and the stator frequency ω_1 is equal to rotor electrical speed $\omega_r = \omega_1 = \text{constant}$:

$$\begin{aligned} V_{A,B,C}(t) &= V\sqrt{2} \cos\left[\omega_1 t - (i-1)\frac{2\pi}{3}\right] \\ I_{A,B,C}(t) &= I\sqrt{2} \cos\left[\omega_1 t - \phi_1 - (i-1)\frac{2\pi}{3}\right] \end{aligned} \quad (5.68)$$

Using the Park transformation with $\theta_{er} = \omega_1 t + \theta_0$ the d - q voltages are obtained:

$$\begin{aligned} V_{d0} &= \frac{2}{3} \left(V_A(t) \cos(-\theta_{er}) + V_B(t) \cos\left(-\theta_{er} + \frac{2\pi}{3}\right) + V_C(t) \cos\left(-\theta_{er} - \frac{2\pi}{3}\right) \right) \\ V_{q0} &= \frac{2}{3} \left(V_A(t) \sin(-\theta_{er}) + V_B(t) \sin\left(-\theta_{er} + \frac{2\pi}{3}\right) + V_C(t) \sin\left(-\theta_{er} - \frac{2\pi}{3}\right) \right) \end{aligned} \quad (5.69)$$

Making use of Equation 5.68 in Equation 5.69 yields the following:

$$\begin{aligned} V_{d0} &= V\sqrt{2} \cos\theta_0 \\ V_{q0} &= -V\sqrt{2} \sin\theta_0 \end{aligned} \quad (5.70)$$

In a similar way, we obtain the currents I_{d0} and I_{q0} :

$$\begin{aligned} I_{d0} &= I\sqrt{2} \cos(\theta_0 + \phi_1) \\ I_{q0} &= -I\sqrt{2} \sin(\theta_0 + \phi_1) \end{aligned} \quad (5.71)$$

Under steady state, the d - q model stator voltages and currents are DC quantities. Consequently, for steady state, we should consider $d/dt = 0$ in Equation 5.67:

$$\begin{aligned} V_{d0} &= \omega_r \Psi_{q0} - I_{d0} r_s; \Psi_{q0} = l_{sl} I_{q0} + l_{qm} I_{q0} \\ V_{q0} &= -\omega_r \Psi_{d0} - I_{q0} r_s; \Psi_{d0} = l_{sl} I_{d0} + l_{dm} (I_{d0} + I_{f0}) \\ V_{f0} &= r_f I_{f0}; \Psi_{f0} = l_{fl} I_{f0} + l_{dm} (I_{d0} + I_{f0}) \\ I_{D0} &= I_{Q0} = 0; \Psi_{D0} = l_{Dl} (I_{d0} + I_{f0}); l_d = l_{dm} + l_{sl} \\ t_e &= -(\Psi_{d0} I_{q0} - \Psi_{q0} I_{d0}); \Psi_{Q0} = l_{qm} I_{q0}; l_q = l_{qm} + l_{sl} \end{aligned} \quad (5.72)$$

Making use of Equation 5.74 in Equation 5.70, we obtain the following:

$$\begin{aligned}
 V_{d0} &= -V\sqrt{2} \sin \delta_{v0} < 0 \\
 V_{q0} &= -V\sqrt{2} \cos \delta_{v0} < 0 \\
 I_{d0} &= -I\sqrt{2} \sin(\delta_{v0} + \varphi_1) < 0 \\
 I_{q0} &= -I\sqrt{2} \cos(\delta_{v0} + \varphi_1) < 0 \text{ for generating}
 \end{aligned} \tag{5.76}$$

The active and reactive powers P_1 and Q_1 are, as expected,

$$\begin{aligned}
 P_1 &= \frac{3}{2}(V_{d0}I_{d0} + V_{q0}I_{q0}) = 3VI \cos \varphi_1 \\
 Q_1 &= \frac{3}{2}(V_{d0}I_{q0} - V_{q0}I_{d0}) = 3VI \sin \varphi_1
 \end{aligned} \tag{5.77}$$

In P.U. quantities, $v_{d0} = -v \times \sin \delta_{v0}$, $v_{q0} = -v \cos \delta_{v0}$, $i_{d0} = -i \sin(\delta_{v0} + \varphi_1)$, and $i_{q0} = -i \cos(\delta_{v0} + \varphi_1)$.

The no-load regime is obtained with $I_{d0} = I_{q0} = 0$, and thus,

$$\begin{aligned}
 V_{d0} &= 0 \\
 V_{q0} &= -\omega_r \Psi_{d0} = -\omega_r l_{dm} I_{f0} = -V / \sqrt{2}
 \end{aligned} \tag{5.78}$$

For no load in Equation 5.74, $\delta_v = 0$ and $I = 0$. V_0 is the no-load phase voltage (RMS value).

For the steady-state short-circuit $V_{d0} = V_{q0} = 0$ in Equation 5.72. If, in addition, $r_s \approx 0$, then $I_{qs} = 0$, and

$$\begin{aligned}
 I_{d0sc} &= \frac{-l_{dm} I_{f0}}{l_d}; \\
 I_{sc3} &= I_{d0sc} / \sqrt{2}
 \end{aligned} \tag{5.79}$$

where I_{sc3} is the phase short-circuit current (RMS value).

Example 5.1

A hydrogenerator with 200 MVA, 24 kV (star connection), 60 Hz, unity power factor, at 90 rpm has the following P.U. parameters: $l_{dm} = 0.6$, $l_{qm} = 0.4$, $l_{sl} = 0.15$, $r_s = 0.003$, $l_{fl} = 0.165$, and $r_f = 0.006$. The field circuit is supplied at 800 V_{dc}. ($V_f = 800$ V).

When the generator works at rated MVA, $\cos \varphi_1 = 1$ and rated terminal voltage, calculate the following:

1. Internal angle δ_{v0}
2. P.U. values of V_{d0} , V_{q0} , I_{d0} , I_{q0}
3. Airgap torque in P.U. quantities and in Nm
4. P.U. field current I_{f0} and its actual value in Amperes

Solution

1. The vector diagram is simplified as $\cos \varphi_1 = 1$ ($\varphi_1 = 0$), but it is worth deriving a formula to directly calculate the power angle δ_{v0} .

Using Equation 5.70 and Equation 5.71 in Equation 5.72 yields the following:

$$\delta_{V0} = \tan^{-1} \left(\frac{\omega_1 l_q I \cos \phi_1 - r_s I \sin \phi_1}{V + r_s I \cos \phi_1 + \omega_1 l_q I \sin \phi_1} \right)$$

with $\phi_1 = 0$ and $\omega_1 = 1$, $I = 1$ P.U. (rated current), and $V = 1$ P.U. (rated voltage):

$$\delta_{V0} = \tan^{-1} \frac{1 \times 0.45 \times 1 \times 1 - 0.0}{1 + 0.003 \times 1 \times 1 \times 0} = 24.16^\circ$$

2. The field current can be calculated from Equation 5.72:

$$i_{f0} = \frac{-V_{q0} - I_{q0} r_s - \omega_r l_d I_{d0}}{\omega_r l_{dm}} = \frac{0.912 + 0.912 \times 0.003 + 1 \cdot (0.6 + 0.15) 0.4093}{1.0 \cdot 0.6} = 2.036 \text{ P.U.}$$

The base current is as follows:

$$I_0 = I_n \sqrt{2} = \frac{S_n \cdot \sqrt{2}}{\sqrt{3} V_{nl}} = \frac{200 \cdot 10^6 \cdot \sqrt{2}}{\sqrt{3} \cdot 24000} = 6792 \text{ A}$$

3. The field circuit P.U. resistance $r_f = 0.006$, and thus, the P.U. field circuit voltage, reduced to the stator is as follows:

$$V'_{f0} = r_f \cdot I_{f0} = 2.036 \times 0.006 = 12.216 \times 10^{-3} \text{ P.U.}$$

Now with $V'_f = 800$ V, the reduction to stator coefficient K_f for field current is

$$K_f = \frac{2}{3} \frac{V'_f}{v_{f0} \cdot V_b} = \frac{2}{3} \frac{800}{12.216 \times 10^{-3} \cdot \frac{24000}{\sqrt{3}} \cdot \sqrt{2}} \approx 2.224$$

Consequently, the field current (in Amperes) I'_{f0} is

$$I'_{f0} = \frac{i_{f0} \cdot I_b}{K_f} = \frac{2.036 \cdot 6792}{2.224} = 6218 \text{ A}$$

So, the excitation power: $P_{exc} = V'_{f0} I'_{f0} = 800 \times 6218 = 4.9744 \text{ MW}$.

4. The P.U. electromagnetic torque is

$$t_e \approx p_e + r_s I^2 = 1.0 + 0.003 \cdot 1^2 = 1.003$$

The torque in Nm is ($2p_1 = 80$ poles) as follows:

$$T_e = t_e \cdot T_{eb} = 1.003 \times \frac{200 \times 10^6}{2\pi \cdot 60 / 40} = 21.295 \times 10^6 \text{ Nm(!)}$$

5.6 The General Equivalent Circuits

Replace d/dt in the P.U. d - q model (Equation 5.67) by using the Laplace operator s/ω_b , which means that the initial conditions are implicitly zero. If they are not, their values should be added.

The general equivalent circuits illustrate Equation 5.67, with d/dt replaced by s/ω_b after separating the main flux linkage components Ψ_{dm} , Ψ_{qm} :

$$\begin{aligned}\Psi_d &= l_{sl} I_d + \Psi_{dm}; \Psi_q = l_{sl} I_q + \Psi_{qm} \\ \Psi_{dm} &= l_{dm} (I_d + I_D + I_f); \Psi_{qm} = l_{qm} (I_q + I_Q) \\ \Psi_f &= l_{fl} I_f + \Psi_{dm}; \Psi_D = l_{sl} I_D + \Psi_{dm} \\ \left(r_0 + \frac{s}{\omega_b} l_0 \right) i_0 &= -V_0\end{aligned}\tag{5.80}$$

with

$$\begin{aligned}\left(r_f + \frac{s}{\omega_b} l_{fl} \right) I_f - V_f &= -\frac{s}{\omega_b} \Psi_{dm} \\ \left(r_D + \frac{s}{\omega_b} l_{Dl} \right) I_D &= -\frac{s}{\omega_b} \Psi_{dm} \\ \left(r_Q + \frac{s}{\omega_b} l_{Ql} \right) I_Q &= -\frac{s}{\omega_b} \Psi_{qm} \\ \left(r_s + \frac{s}{\omega_b} l_{sl} \right) I_d + V_d - \omega_r \Psi_q &= -\frac{s}{\omega_b} \Psi_{dm} \\ \left(r_s + \frac{s}{\omega_b} l_{sl} \right) I_q + V_q + \omega_r \Psi_d &= -\frac{s}{\omega_b} \Psi_{qm}\end{aligned}\tag{5.81}$$

Equation 5.81 evidentiates three circuits in parallel along axis d and two equivalent circuits along axis q . It is also implicit that the coupling of the circuits along axis d and q is performed only through the main flux components Ψ_{dm} and Ψ_{qm} . Magnetic saturation and frequency effects are not yet considered.

Based on Equation 5.81, the general equivalent circuits of SG are shown in [Figure 5.6a](#) and [Figure 5.6b](#). A few remarks on [Figure 5.6](#) are as follows:

- The magnetization current components I_{dm} and I_{qm} are defined as the sum of the d - q model currents:

$$\begin{aligned}I_{dm} &= I_d + I_D + I_f; \\ I_{qm} &= I_q + I_Q\end{aligned}\tag{5.82}$$

- There is no magnetic coupling between the orthogonal axes d and q , because magnetic saturation is either ignored or considered separately along each axis as follows:

$$l_{sl}(i_s); l_{dm}(I_{dm}); l_{sl}(I_s), l_{qm}(I_{qm}); I_s = \sqrt{I_d^2 + I_q^2}$$

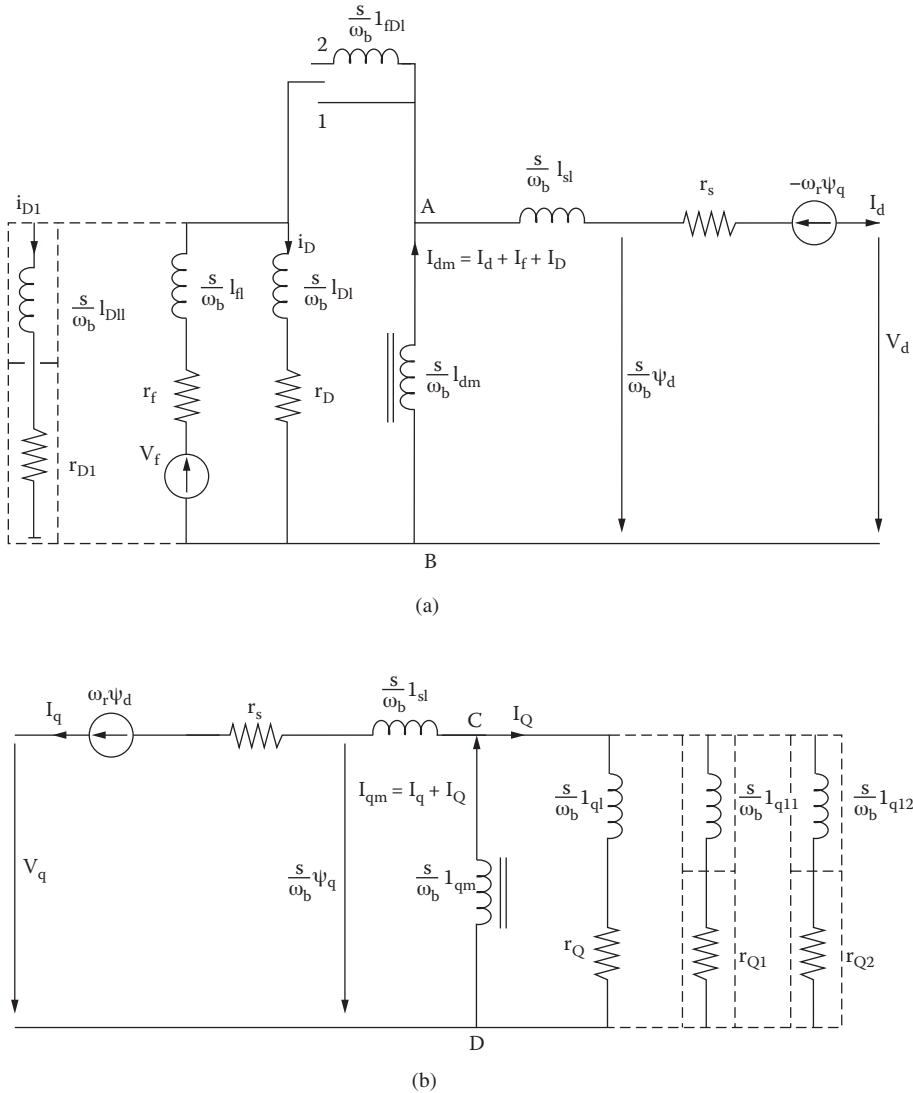


FIGURE 5.6 General equivalent circuits of synchronous generators: (a) along axis d and (b) along axis q .

- Should the frequency (skin) effect be present in the rotor damper cage (or in the rotor pole solid iron), additional rotor circuits are added in parallel. In general, one additional circuit along axis d and two along axis q are sufficient even for solid rotor pole SGs (Figure 5.6a and Figure 5.6b). In these cases, additional equations have to be added to Equation 5.81, but their composure is straightforward.
- Figure 5.6a also exhibits the possibility of considering the additional, leakage type, flux linkage (inductance, l_{fl}) between the field and damper cage windings, in salient pole rotors. This inductance is considered instrumental when the field-winding parameter identification is checked after the stator parameters were estimated in tests with measured stator variables. Sometimes, l_{fl} is estimated as negative.
- For steady state, $s = 0$ in the equivalent circuits, and thus, the voltages V_{AB} and V_{CD} are zero. Consequently, $I_{D0} = I_{Q0} = 0$, $V_{f0} = -r_f I_{f0}$ and the steady state d - q model equations may be “read” from Figure 5.6a and Figure 5.6b.

- The null component voltage equation in Equation 5.80:

$$V_0 + \left(r_0 + \frac{s}{\omega_b} l_0 \right) i_0 = 0$$

does not appear, as expected, in the general equivalent circuit because it does not interfere with the main flux fundamental. In reality, the null component may produce some eddy currents in the rotor cage through its third space-harmonic mmf.

5.7 Magnetic Saturation Inclusion in the d - q Model

The magnetic saturation level is, in general, different in various regions of an SG cross-section. Also, the distribution of the flux density in the airgap is not quite sinusoidal. However, in the d - q model, only the flux-density fundamental is considered. Further, the leakage flux path saturation is influenced by the main flux path saturation. A realistic model of saturation would mean that all leakage and main inductances depend on all currents in the d - q model. However, such a model would be too cumbersome to be practical.

Consequently, we will present here only two main approximations of magnetic saturation inclusion in the d - q model from the many proposed so far [2–7]. These two appear to us to be representative. Both include cross-coupling between the two orthogonal axes due to main flux path saturation. While the first presupposes the existence of a unique magnetization curve along axes d and q , respectively, in relation to total mmf ($I_m = \sqrt{I_{dm}^2 + I_{qm}^2}$), the second curve fits the family of curves $\Psi_{dm}^*(I_{dm}, I_{qm})$, $\Psi_{qm}^*(I_{dm}, I_{qm})$, keeping the dependence on both I_{dm} and I_{qm} .

In both models, the leakage flux path saturation is considered separately by defining transient leakage inductances $l_{sl}^t, l_{Dl}^t, l_{fl}^t$:

$$\begin{aligned} l_{sl}^t &= l_{sl} + \frac{\partial l_{sl}}{\partial i_s} i_s \leq l_{sl}; I_s = \sqrt{I_d^2 + I_q^2} \\ l_{Dl}^t &= l_{Dl} + \frac{\partial l_{Dl}}{\partial i_D} i_D < l_{Dl} \\ l_{fl}^t &= l_{fl} + \frac{\partial l_{fl}}{\partial i_f} i_f < l_{fl} \\ l_{Ql}^t &= l_{Ql} + \frac{\partial l_{Ql}}{\partial i_Q} i_Q < l_{Ql} \end{aligned} \tag{5.83}$$

Each of the transient inductances in Equation 5.83 is considered as being dependent on the respective current.

5.7.1 The Single d - q Magnetization Curves Model

According to this model of main flux path saturation, the distinct magnetization curves along axes d and q depend only on the total magnetization current I_m [2, 3].

$$\begin{aligned} \Psi_{dm}^*(I_m) &\neq \Psi_{qm}^*(I_m); I_m = \sqrt{I_{dm}^2 + I_{qm}^2} \\ I_{dm} &= I_d + I_D + I_f \\ I_{qm} &= I_q + I_Q \end{aligned} \tag{5.84}$$

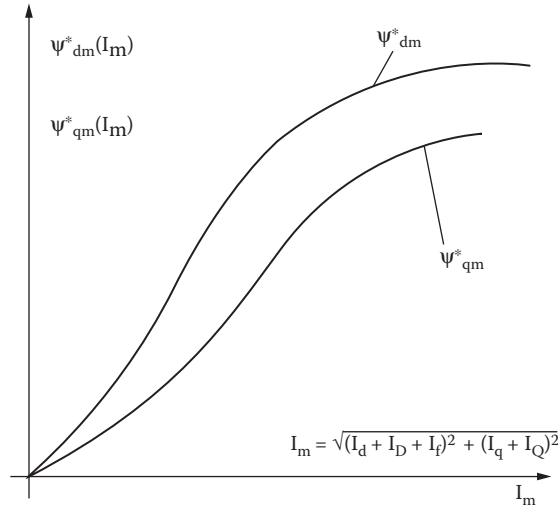


FIGURE 5.7 The unique d - q magnetization curves.

Note that the two distinct, but unique, d and q axes magnetization curves shown in Figure 5.7 represent a disputable approximation. It is only recently that finite element method (FEM) investigations showed that the concept of unique magnetization curves does not hold with the SG for underexcited (draining reactive power) conditions [4]: $I_m < 0.7$ P.U. For $I_m > 0.7$, the model apparently works well for a wide range of active and reactive power load conditions. The magnetization inductances l_{dm} and l_{qm} are also functions of I_m , only

$$\begin{aligned}\Psi_{dm} &= l_{dm}(I_m) \cdot I_{dm} \\ \Psi_{qm} &= l_{qm}(I_m) \cdot I_{qm}\end{aligned}\tag{5.85}$$

with

$$\begin{aligned}l_{dm}(I_m) &= \frac{\Psi_{dm}^*(I_m)}{I_m} \\ l_{qm}(I_m) &= \frac{\Psi_{qm}^*(I_m)}{I_m}\end{aligned}\tag{5.86}$$

Notice that the $\Psi_{dm}^*(I_m), \Psi_{qm}^*(I_m)$ may be obtained through tests where either only one or both components (I_{dm}, I_{qm}) of magnetization current I_m are present. This detail should not be overlooked if coherent results are to be expected. It is advisable to use a few combinations of I_{dm} and I_{qm} for each axis and use curve-fitting methods to derive the unique magnetization curves $\Psi_{dm}^*(I_m), \Psi_{qm}^*(I_m)$. Based on Equation 5.85 and Equation 5.86, the main flux time derivatives are obtained:

$$\begin{aligned}\frac{d\Psi_{dm}}{dt} &= \frac{d\Psi_{dm}^*}{dI_m} \frac{dI_m}{dt} \cdot \frac{I_{dm}}{I_m} + \frac{\Psi_{dm}^*}{I_m^2} \left(I_m \frac{dI_{dm}}{dt} - I_{dm} \frac{dI_m}{dt} \right) \\ \frac{d\Psi_{qm}}{dt} &= \frac{d\Psi_{qm}^*}{dI_m} \frac{dI_m}{dt} \cdot \frac{I_{qm}}{I_m} + \frac{\Psi_{qm}^*}{I_m^2} \left(I_m \frac{dI_{qm}}{dt} - I_{qm} \frac{dI_m}{dt} \right)\end{aligned}\tag{5.87}$$

with

$$\frac{dI_m}{dt} = \frac{I_{qm}}{I_m} \frac{dI_{qm}}{dt} + \frac{I_{dm}}{I_m} \frac{dI_{dm}}{dt} \quad (5.88)$$

Finally,

$$\begin{aligned} \frac{d\Psi_{dm}}{dt} &= l_{ddm} \frac{dI_{dm}}{dt} + l_{qdm} \frac{dI_{qm}}{dt} \\ \frac{d\Psi_{qm}}{dt} &= l_{dqm} \frac{dI_{dm}}{dt} + l_{qqm} \frac{dI_{qm}}{dt} \end{aligned} \quad (5.89)$$

$$\begin{aligned} l_{ddm} &= l_{dmt} \frac{I_{dm}^2}{I_m^2} + l_{dm} \frac{I_{qm}^2}{I_m^2} \\ l_{qqm} &= l_{qmt} \frac{I_{qm}^2}{I_m^2} + l_{qm} \frac{I_{dm}^2}{I_m^2} \end{aligned} \quad (5.90)$$

$$l_{dqm} = l_{qdm} = (l_{dmt} - l_{dm}) I_{dm} \frac{I_{qm}}{I_m^2} \quad (5.91)$$

$$l_{dmt} - l_{dm} = l_{qmt} - l_{qm}$$

$$\begin{aligned} l_{dmt} &= \frac{d\Psi_{dm}^*}{dI_m} \\ l_{qmt} &= \frac{d\Psi_{qm}^*}{dI_m} \end{aligned} \quad (5.92)$$

The equality of coupling transient inductances $l_{dqm} = l_{qdm}$ between the two axes is based on the reciprocity theorem. l_{dmt} and l_{qmt} are the so-called differential d and q axes magnetization inductances, while l_{ddm} and l_{qqm} are the transient magnetization self-inductances with saturation included. All of these inductances depend on both I_{dm} and I_{qm} , while l_{dm} , l_{dmt} , l_{qm} , l_{qmt} depend only on I_m .

For the situation when DC premagnetization occurs, the differential magnetization inductances l_{dmt} and l_{qmt} should be replaced by the so-called incremental inductances l_{dm}^i , l_{qm}^i :

$$\begin{aligned} l_{dm}^i &= \frac{\Delta\Psi_{dm}^*}{\Delta I_m} \\ l_{qm}^i &= \frac{\Delta\Psi_{qm}^*}{\Delta I_m} \end{aligned} \quad (5.93)$$

l_{dm}^i and l_{qm}^i are related to the incremental permeability in the iron core when a superposition of DC and alternating current (AC) magnetization occurs (Figure 5.8).

The normal permeability of iron $\mu_n = B_m/H_m$ is used when calculating the magnetization inductances l_{dm} and l_{qm} ; $\mu_d = dB_m/dH_m$ for l_{dm}^i and l_{qm}^i , and $\mu_i = \Delta B_m/\Delta H_m$ (Figure 5.8) for the incremental magnetization inductances l_{md}^i and l_{mq}^i .

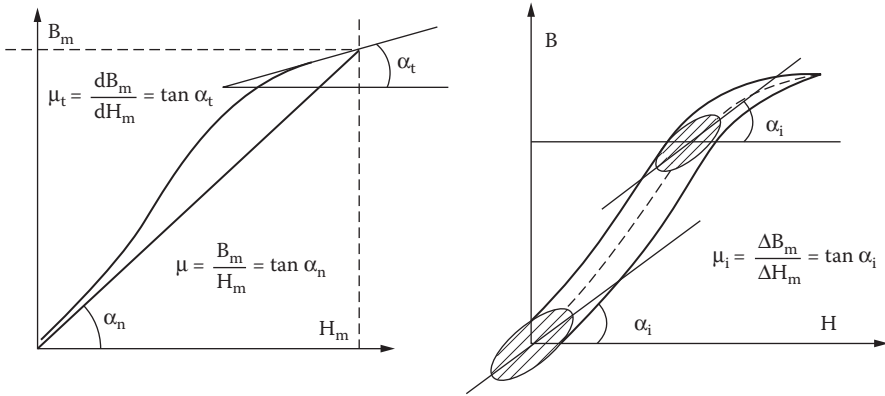


FIGURE 5.8 Iron permeabilities.

For the incremental inductances, the permeability μ_i corresponds to a local small hysteresis cycle (in Figure 5.8), and thus, $\mu_i < \mu_d < \mu_n$. For zero DC premagnetization and small AC voltages (currents) at standstill, for example, $\mu_i \approx (100 - 150) \mu_0$, which explains why the magnetization inductances correspond to l_{md}^i and l_{mq}^i rather than to l_{dm} and l_{qm} and are much smaller than the latter (Figure 5.9).

Once $l_{dm}^i(I_m)$, $l_{qm}^i(I_m)$, $l_{dm}^t(I_m)$, $l_{qm}^t(I_m)$, $l_{dm}^i(I_m)$, $l_{qm}^i(I_m)$ are determined, either through field analysis or through experiments, then $l_{ddm}^i(I_{dm}, I_{qm})$, $l_{qqm}^i(I_{dm}, I_{qm})$ may be calculated with I_{dm} and I_{qm} given. Interpolation through tables or analytical curve fitting may be applied to produce easy-to-use expressions for digital simulations.

The single unique d - q magnetization curves model included the cross-coupling implicitly in the expressions of Ψ_{dm} and Ψ_{qm} , but it considers it explicitly in the $d\Psi_{dm}/dt$ and $d\Psi_{qm}/dt$ expressions, that is, in the transients. Either with currents I_{dm} , I_{qm} , I_f , I_D , I_Q , ω_r , θ_{er} or with flux linkages Ψ_{dm} , Ψ_{qm} , Ψ_f , Ψ_D , Ψ_Q , ω_r , θ_{er} (or with quite a few intermediary current, flux-linkage combinations) as variables, models based on the same concepts may be developed and used rather handily for the study of both steady states and transients [5]. The computation of $\Psi_{dm}^*(I_m)$, $\Psi_{qm}^*(I_m)$ functions or their measurement from standstill tests is straightforward.

This tempting simplicity is paid for by the limitation that the unique d - q magnetization curves concept does not seem to hold when the machine is notably underexcited, with the emf lower than the terminal voltage, because the saturation level is smaller despite the fact that I_m is about the same as that for the lagging power factor at constant voltage [4].

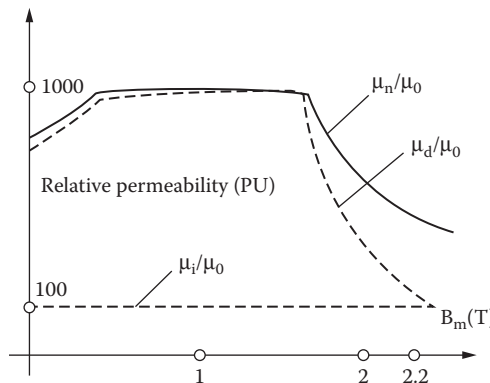


FIGURE 5.9 Typical per unit (P.U.) normal, differential, and incremental permeabilities of silicon laminations.

This limitation justifies the search for a more general model that is valid for the whole range of the active (reactive) power capability envelope of the SG. We call this the multiple magnetization curve model.

5.7.2 The Multiple d - q Magnetization Curves Model

This kind of model presupposes that the d and q axes flux linkages Ψ_d and Ψ_q are explicit functions of I_d , I_q , I_{dm} , I_{qm} :

$$\begin{aligned}
 \Psi_d &= l_{sl}I_d + l_{dq}(I_q + I_Q) + l_{dm}(I_f + I_D + I_d) = l_{sl}I_d + l_{dq}I_{qm} + l_{dms}I_{dm} = l_{sl}I_d + \Psi_{dms} \\
 \Psi_q &= l_{sl}I_d + l_{qm}(I_q + I_Q) + l_{dq}(I_d + I_D + I_f) = l_{sl}I_q + l_{qms}I_{qm} + l_{dq}I_{dm} = l_{sl}I_q + \Psi_{qms} \\
 \Psi_f &= l_{fl}I_f + l_{dm}(I_f + I_D + I_d) + l_{dq}(I_q + I_Q) = l_{fl}I_f + l_{dms}I_{dm} + l_{dq}I_{qm} = l_{fl}I_f + \Psi_{dms} \\
 \Psi_D &= l_{Dl}I_D + l_{dm}(I_f + I_D + I_d) + l_{dq}(I_q + I_Q) = l_{Dl}I_D + l_{dms}I_{dm} + l_{dq}I_{qm} = l_{Dl}I_D + \Psi_{dms} \\
 \Psi_Q &= l_{Ql}I_Q + l_{dq}(I_f + I_D + I_d) + l_{qm}(I_q + I_Q) = l_{Ql}I_Q + l_{dq}I_{dm} + l_{qms}I_{qm} = l_{Ql}I_Q + \Psi_{qms}
 \end{aligned} \tag{5.94}$$

Now, l_{dms} and l_{qms} are functions of both I_{dm} and I_{qm} . Conversely, $\Psi_{dms}(I_{dm}, I_{qm})$ and $\Psi_{qms}(I_{dm}, I_{qm})$ are two families of magnetization curves that have to be found either by computation or by experiments.

For steady state, $I_D = I_Q = 0$, but otherwise, Equation 5.94 holds. Basically, the Ψ_{dms} and Ψ_{qms} curves look like as shown in Figure 5.10:

$$\begin{aligned}
 \Psi_{dms} &= l_{dms}(I_{dm}, I_{qm})I_{dm} \\
 \Psi_{qms} &= l_{qms}(I_{dm}, I_{qm})I_{qm}
 \end{aligned} \tag{5.95}$$

Once this family of curves is acquired (by FEM analysis or by experiments), various analytical approximations may be used to curve-fit them adequately.

Then, with Ψ_d , Ψ_q , Ψ_f , Ψ_D , Ψ_Q , ω_r , and θ_{er} as variables and I_f , I_D , I_Q , I_d , and I_q as dummy variables, the $\Psi_{dms}(I_{dm}, I_{qm})$, $\Psi_{qms}(I_{dm}, I_{qm})$ functions are used in Equation 5.94 to calculate iteratively each time step, the dummy variables.

When using flux linkages as variables, no additional inductances responsible for cross-coupling magnetic saturation need to be considered. As they are not constant, their introduction does not seem practical. However, such attempts keep reoccurring [6, 7], as the problem seems far from a definitive solution.

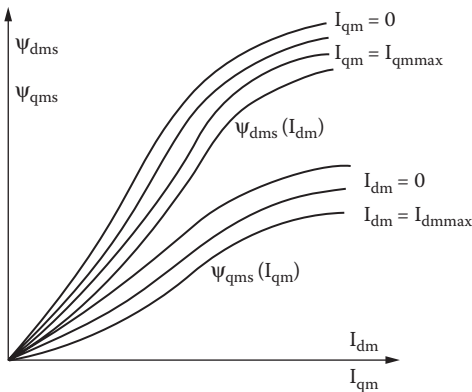


FIGURE 5.10 Family of magnetization curves.

Considering cross-coupling due to magnetic saturation seems to be necessary when calculating the field current, stator current, and power angle, under steady state for given active and reactive power and voltage, with an error less than 2% for the currents and around a 1° error for the power angle [4, 7]. Also, during large disturbance transients, where the main flux varies notably, the cross-coupling saturation effect is to be considered.

Though magnetic saturation is very important for refined steady state and for transient investigations, most of the theory of transients for the control of SGs is developed for constant parameter conditions — operational parameters is such a case.

5.8 The Operational Parameters

In the absence of magnetic saturation variation, the general equivalent circuits of SG (Figure 5.6) lead to the following generic expressions of operational parameters in the Ψ_d and Ψ_q operational expressions:

$$\begin{aligned}\Psi_d(s) &= l_d(s) \cdot I_d(s) + g(s) v_{ex}(s) \quad [P.U.] \\ \Psi_q(s) &= l_q(s) \cdot I_q(s)\end{aligned}\tag{5.96}$$

with

$$\begin{aligned}l_d(s) &= \frac{(1 + sT_d') (1 + sT_d'')}{(1 + sT_{d0}') (1 + sT_{d0}'')} \cdot l_d \quad [P.U.] \\ l_q(s) &= \frac{(1 + sT_q'')}{(1 + sT_{q0}'')} \cdot l_q \quad [P.U.] \\ g(s) &= \frac{(1 + sT_D)}{(1 + sT_{d0}') (1 + sT_{d0}'')} \cdot \frac{l_{dm}}{r_f} [P.U.]\end{aligned}\tag{5.97}$$

Sparing the analytical derivations, the time constants $T_d', T_d'', T_{d0}', T_{d0}'', T_q', T_{q0}''$ have the following expressions:

$$\begin{aligned}T_d' &\approx \frac{1}{\omega_b r_f} \left(l_{fl} + l_{fdl} + \frac{l_{dm} l_{sl}}{l_{dm} + l_{sl}} \right); [s] \\ T_d'' &\approx \frac{1}{\omega_b r_D} \left(l_{Dl} + \frac{l_{dm} l_{fdl} l_{fl} + l_{dm} l_{sl} l_{fl} + l_{sl} l_{fdl} l_{fl}}{l_{dm} l_{fl} + l_{fl} l_{sl} + l_{dm} l_{fdl} + l_{sl} l_{fdl} + l_{fdl} l_{fl} + l_{dm} l_{sl}} \right); [s] \\ T_{d0}' &\approx \frac{1}{\omega_b r_f} (l_{dm} + l_{fdl} + l_{fl}); [s] \\ T_{d0}'' &\approx \frac{1}{\omega_b r_D} \left(l_{Dl} + \frac{l_{fl} (l_{dm} + l_{fdl})}{l_{fl} + l_{dm} + l_{fdl}} \right); [s] \\ T_q'' &\approx \frac{1}{\omega_b r_Q} \left(l_{ql} + \frac{l_{qm} l_{sl}}{l_{qm} + l_{sl}} \right); [s]\end{aligned}\tag{5.98}$$

$$T_{q0}'' \approx \frac{1}{\omega_b r_Q} (l_{Ql} + l_{qm}); [s]$$

$$T_D \approx \frac{l_{Dl}}{\omega_b r_D}; [s]$$
(5.98 cont.)

As already mentioned, with ω_b measured in rad/sec, the time constants are all in seconds, while all resistances and inductances are in P.U. values. The time constants differ between each other up to more than 100-to-1 ratios. T_{d0}' is of the order of seconds in large SGs, while T_d' , T_{d0}'' , T_{q0}'' are in the order of a few tenths of a second, T_d'' , T_q'' in the order of a few tenths of milliseconds, and T_D in the order of a few milliseconds.

Such a broad spectrum of time constants indicates that the SG equations for transients (Equation 5.81) represent a stiff system. Consequently, the solution through numerical methods needs time integration steps smaller than the lowest time constant in order to correctly portray all occurring transients. The above time constants are catalog data for SGs:

- T_{d0}' : d axis open circuit field winding (transient) time constant ($I_d = 0$, $I_D = 0$)
- T_{d0}'' : d axis open circuit damper winding (subtransient) time constant ($I_d = 0$)
- T_d' : d axis transient time constant ($I_D = 0$) — field-winding time constant with short-circuited stator but with open damper winding
- T_{d0}' : d axis subtransient time constant — damper winding time constant with short-circuited field winding and stator
- T_{q0}'' : q axis open circuit damper winding (subtransient) time constant ($I_q = 0$)
- T_q'' : q axis subtransient time constant (q axis damper winding time constant with short-circuited stator)
- T_D : d axis damper winding self-leakage time constant

In the industrial practice of SGs, the limit — initial and final — values of operational inductances have become catalog data:

$$l_d'' = \lim_{\substack{s \rightarrow \infty \\ (t \rightarrow 0)}} l_d(s) = l_d \cdot \frac{T_d' T_d''}{T_{d0}' T_{d0}''}$$

$$l_d' = \lim_{\substack{s \rightarrow \infty \\ T_d'' = T_{d0}'' = 0}} l_d(s) = l_d \cdot \frac{T_d'}{T_{d0}'}$$

$$l_d = \lim_{\substack{s \rightarrow 0 \\ t \rightarrow \infty}} l_d(s) = l_d \quad (5.99)$$

$$l_q'' = \lim_{\substack{s \rightarrow \infty \\ (t \rightarrow 0)}} l_q(s) = l_q \cdot \frac{T_q''}{T_{q0}''}$$

$$l_q = \lim_{\substack{s \rightarrow 0 \\ t \rightarrow \infty}} l_q(s) = l_q$$

where

- l_d'' , l_d' , l_d = the d axis subtransient, transient, and synchronous inductances
- l_q'' , l_q = the q axis subtransient and synchronous inductances
- l_p = the Potier inductance in P.U. ($l_p \geq l_{sl}$)

Typical values of the time constants (in seconds) and subtransient and transient and synchronous inductances (in P.U.) are shown in [Table 5.1](#).

As Table 5.1 suggests, various inductances and time constants that characterize the SG are constants. In reality, they depend on magnetic saturation and skin effects (in solid rotors), as suggested in previous

TABLE 5.1 Typical Synchronous Generator Parameter Values

Parameter	Two-Pole Turbogenerator	Hydrogenerators
I_d (P.U.)	0.9–1.5	0.6–1.5
I_q (P.U.)	0.85–1.45	0.4–1.0
I_d' (P.U.)	0.12–0.2	0.2–0.5
I_d'' (P.U.)	0.07–0.14	0.13–0.35
I_{fd} (P.U.)	0.05–+0.05	0.05–+0.05
I_0 (P.U.)	0.02–0.08	0.02–0.2
I_p (P.U.)	0.07–0.14	0.15–0.2
r_s (P.U.)	0.0015–0.005	0.002–0.02
T_{d0} (sec)	2.8–6.2	1.5–9.5
T_d' (sec)	0.35–0.9	0.5–3.3
T_d'' (sec)	0.02–0.05	0.01–0.05
T_{d0}'' (sec)	0.02–0.15	0.01–0.15
T_q'' (sec)	0.015–0.04	0.02–0.06
T_{q0}'' (sec)	0.04–0.08	0.05–0.09
I_q'' (P.U.)		0.2–0.45

Note: P.U. stands for per unit; sec stands for second(s).

paragraphs. There are, however, transient regimes where the magnetic saturation stays practically the same, as it corresponds to small disturbance transients. On the other hand, in high-frequency transients, the I_d and I_q variation with magnetic saturation level is less important, while the leakage flux paths saturation becomes notable for large values of stator and rotor current (the beginning of a sudden short-circuit transient).

To make the treatment of transients easier to approach, we distinguish here a few types of transients:

- Fast (electromagnetic) transients: speed is constant
- Electromechanical transients: electromagnetic + mechanical transients (speed varies)
- Slow (mechanical) transients: electromagnetic steady state; speed varies

In what follows, we will treat each of these transients in some detail.

5.9 Electromagnetic Transients

In fast (electromagnetic) transients, the speed may be considered constant; thus, the equation of motion is ignored. The stator voltage equations of Equation 5.81 in Laplace form with Equation 5.96 become the following:

$$\begin{aligned}
 -v_d(s) &= r_s I_d + \frac{s}{\omega_b} I_d(s) I_d - \omega_r I_q(s) I_q + g(s) \frac{s}{\omega_b} v_f(s) \\
 -v_q(s) &= r_s I_q + \frac{s}{\omega_b} I_q(s) I_q + \omega_r [I_d(s) I_d + g(s) v_f(s)]
 \end{aligned}
 \tag{5.100}$$

Note that ω_r is in relative units, and for rated rotor speed, $\omega_r = 1$.

If the initial values I_{d0} and I_{q0} of variables I_d and I_q are known and the time variation of $v_d(t)$, $v_q(t)$, and $v_f(t)$ may be translated into Laplace forms of $v_d(s)$, $v_q(s)$, and $v_f(s)$, then Equation 5.100 may be solved to obtain the $i_d(s)$ and $i_q(s)$:

$$\begin{vmatrix} -v_d(s) & -g(s) \frac{s}{\omega_b} v_f(s) \\ -v_q(s) & -\omega_r g(s) v_f(s) \end{vmatrix} = \begin{vmatrix} r_s + \frac{s I_d(s)}{\omega_b} & -\omega_r I_q(s) \\ \omega_r I_d(s) & r_s + \frac{s}{\omega_b} I_q(s) \end{vmatrix} \begin{vmatrix} i_d(s) \\ i_q(s) \end{vmatrix}
 \tag{5.101}$$

Though $I_d(s)$ and $I_q(s)$ may be directly derived from Equation 5.101, their expressions are hardly practical in the general case.

However, there are a few particular operation modes where their pursuit is important. The sudden three-phase short-circuit from no load and the step voltage or AC operation at standstill are considered here. To start, the voltage buildup at no load, in the absence of a damper winding, is treated.

Example 5.2: The Voltage Buildup at No Load

Apply Equation 5.101 for the stator voltage buildup at no load in an SG without a damper cage on the rotor when the 100% step DC voltage is applied to the field winding.

Solution

With I_d and I_q being zero, what remains from Equation 5.101 is as follows:

$$\begin{aligned} -v_d(s) &= g(s) \frac{s}{\omega_b} v_f(s) \\ -v_q(s) &= \omega_r \cdot g(s) \cdot v_f(s) \end{aligned} \quad (5.102a)$$

The Laplace transform of a step function is applied to the field-winding terminals:

$$v_f(s) = \frac{v_f}{s} \quad (5.102b)$$

The transfer function $g(s)$ from Equation 5.97, with $l_{Dl} = l_{fl} = 0$, $v_D = 0$, and zero stator currents, is as follows:

$$g(s) = \frac{l_{dm}}{r_f} \times \frac{1}{1 + sT'_{d0}} \quad (5.103)$$

$$T'_{d0} = \frac{l_{dm} + l_{fl}}{r_f \cdot \omega_b} \quad (5.104)$$

Finally,

$$v_d(t) = -v_f \frac{l_{dm}}{(l_{dm} + l_{fl})} e^{-\frac{t}{T'_{d0}}} \quad (5.105)$$

$$v_q(t) = -v_f \frac{\omega_r l_{dm}}{r_f} e^{-\frac{t}{T'_{d0}}} \quad (5.106)$$

with $\omega_r = 1$ P.U., $l_{dm} = 1.2$ P.U., $l_{fl} = 0.2$ P.U., $r_f = 0.003$ P.U., $v_{f0} = 0.003$ P.U., and $\omega_b = 2 \times 60 \times \pi = 377$ rad/sec:

$$T'_{d0} = \frac{1.2 + 0.2}{0.003 \cdot 377} = 1.2378 \text{ sec}$$

$$v_d(t) = -0.003 \frac{1.2}{1.2 + 0.2} e^{-\frac{t}{1.2378}} = -2.5714 \times 10^{-3} \cdot e^{-t/1.2378} [P.U.]$$

$$v_q(t) = -0.003 \times 1 \times \frac{1.2}{0.003} \left(1 - e^{-\frac{t}{1.2378}} \right) = -1.2 \left(1 - e^{-t/1.2378} \right) [P.U.]$$

The phase voltage of phase A is (Equation 5.30)

$$\begin{aligned} v_A(t) &= v_d(t) \cos \omega_b t - v_q(t) \sin \omega_b t = \\ &= -2.5714 \times 10^{-3} e^{-t/1.2378} \cos(\omega_b t + \theta_0) + 1.2 \left(1 - e^{-t/1.2378} \right) \sin(\omega_b t + \theta_0) [P.U.] \end{aligned}$$

For no load, from Equation 5.75, with zero power angle ($\delta_{v0} = 0$),

$$\theta_0 = -\frac{3\pi}{2}$$

In a similar way, $v_b(t)$ and $v_c(t)$ are obtained using Park inverse transformation.

The stator symmetrical phase voltages may be expressed simply in volts by multiplying the voltages in P.U. to the base voltage $V_b = V_n \times \sqrt{2}$; V_n is the base RMS phase voltage.

5.10 The Sudden Three-Phase Short-Circuit from No Load

The initial no-load conditions are characterized by $I_{d0} = I_{q0} = 0$. Also, if the field-winding terminal voltage is constant,

$$v_{f0} = I_{f0} \cdot r_f \quad (5.107)$$

From Equation 5.101, this time with $s = 0$ and $I_{d0} = I_{q0} = 0$, it follows that

$$(v_{d0})_{s=0} = 0 \quad (5.108)$$

$$(v_{q0})_{s=0} = -\omega_{r0} \frac{I_{dm}}{r_f} v_{f0}$$

So, already for the initial conditions, the voltage along axis d , v_{d0} , is zero under no load. For axis q , the no-load voltage occurs. To short-circuit the machine, we simply have to apply along axis q the opposite voltage $-v_{q0}$.

Notice that, as $v_f = v_{f0}$, $v_f(s) = 0$, Equation 5.101 becomes as follows:

$$\begin{vmatrix} 0 \\ -\frac{v_{q0} \cdot \omega_b}{s} \end{vmatrix} = \begin{vmatrix} r_s + \frac{s}{\omega_b} I_d(s) & -\omega_{r0} I_q(s) \\ \omega_{r0} I_d(s) & r_0 + \frac{s}{\omega_b} I_q(s) \end{vmatrix} \cdot \begin{vmatrix} I_d(s) \\ I_q(s) \end{vmatrix} \quad (5.109)$$

The solution of Equation 5.110 is straightforward, with

$$I_d^*(s) = \frac{-v_{q0}\omega_b^3\omega_r}{sl_d(s)\left[\omega_b^2\omega_r^2 + s^2 + sr_s\omega_b\left(\frac{1}{l_d(s)} + \frac{1}{l_q(s)}\right) + \frac{r_s^2\omega_b^2}{l_d(s)\cdot l_q(s)}\right]} \quad (5.110)$$

As it is, $I_d(s)$ would be difficult to handle, so two approximations are made: the terms in r_s^2 are neglected, and

$$\frac{r_s\omega_b}{2}\left(\frac{1}{l_d(s)} + \frac{1}{l_q(s)}\right) \approx \frac{1}{T_a} = \text{const.} \quad (5.111)$$

with

$$\frac{1}{T_a} \approx \frac{r_s\omega_b}{2}\left(\frac{1}{l_d''} + \frac{1}{l_q''}\right) \quad (5.112)$$

With Equation 5.111 and Equation 5.112, Equation 5.110' becomes

$$I_d(s) \approx \frac{-V_{q0}\omega_b^3\omega_r}{s\left(s^2 + \frac{2}{T_a}s + \omega_b^2\omega_r^2\right)} \cdot \frac{1}{l_d(s)} \quad (5.113)$$

$$I_q(s) \approx \frac{-V_{q0}\omega_b^2\omega_r}{\left(s^2 + \frac{2}{T_a}s + \omega_b^2\omega_r^2\right)} \cdot \frac{1}{l_q(s)} \quad (5.114)$$

Making use of Equation 5.96 and Equation 5.97, $1/l_d(s)$ and $1/l_q(s)$ may be expressed as follows:

$$\frac{1}{l_d(s)} = \frac{1}{l_d} + \left(\frac{1}{l_d'} - \frac{1}{l_d}\right) \frac{s}{s+1/T_d'} + \left(\frac{1}{l_d''} - \frac{1}{l_d'}\right) \frac{s}{s+1/T_d''} \quad (5.115)$$

$$\frac{1}{l_q(s)} = \frac{1}{l_q} + \left(\frac{1}{l_q'} - \frac{1}{l_q}\right) \frac{s}{s+1/T_q'} + \left(\frac{1}{l_q''} - \frac{1}{l_q'}\right) \frac{s}{s+1/T_q''} \quad (5.116)$$

With T_d' , T_q'' larger than $1/\omega_b$ and $\omega_r = 1.0$, after some analytical derivations with approximations, the inverse Laplace transforms of $I_d(s)$ and $I_q(s)$ are obtained:

$$I_d(t) \approx -v_{q0}\left[\frac{1}{l_d} + \left(\frac{1}{l_d'} - \frac{1}{l_d}\right)e^{-t/T_d'} + \left(\frac{1}{l_d''} - \frac{1}{l_d'}\right)e^{-t/T_d''} - \frac{1}{l_d''}e^{-t/T_a}\cos\omega_b t\right]$$

$$I_q(t) \approx -\frac{v_{q0}}{l_q''}e^{-t/T_a}\sin\omega_b t; \quad (5.117)$$

$$I_d(t) = I_{d0} + I_d(t)$$

$$I_q(t) = I_{q0} + I_q(t)$$

The sudden phase short-circuit current from no load ($I_{d0} = I_{q0} = 0$) is obtained, making use of the following:

$$\begin{aligned}
 I_A(t) = & \left[I_d(t) \cos(\omega_b t + \gamma_0) - I_q(t) \sin(\omega_b t + \gamma_0) \right] = \\
 & -v_{q0} \left\{ \left[\frac{1}{l_d} + \left(\frac{1}{l_d'} - \frac{1}{l_d} \right) e^{-t/T_d'} + \left(\frac{1}{l_d''} - \frac{1}{l_d'} \right) e^{-t/T_d''} \right] \cos(\omega_b t + \gamma_0) \right. \\
 & \left. - \frac{1}{2} \left(\frac{1}{l_d''} - \frac{1}{l_q''} \right) e^{-t/T_a} - \frac{1}{2} \left(\frac{1}{l_d''} - \frac{1}{l_q''} \right) e^{-t/T_a} \cos(2\omega_b t + \gamma_0) \right\}
 \end{aligned} \quad (5.118)$$

The relationship between $I_f(s)$ and $I_d(s)$ for the case in point ($v_f(s) = 0$) is

$$I_f(s) = -g(s) \frac{s}{\omega_b} I_d(s) \quad (5.119)$$

Finally,

$$I_f(t) = I_{f0} + I_{f0} \cdot \frac{(l_d - l_d')}{l_d} \left[e^{-t/T_d'} - \left(1 - T_D / T_D'' \right) e^{-t/T_d''} - \frac{T_D}{T_d''} e^{-t/T_a} \cos \omega_b t \right] \quad (5.120)$$

Typical sudden short-circuit currents are shown in [Figure 5.11](#) (parts a, b, c, and d). Further, the flux linkages $\Psi_d(s)$, $\Psi_q(s)$ (with $v_f(s) = 0$) are as follows:

$$\begin{aligned}
 \Psi_d(s) = l_d(s) I_d(s) &= \frac{-\omega_b^3 \omega_r^2}{\left(s^2 + \frac{2s}{T_a} + \omega_b^2 \omega_r^2 \right)} \cdot \frac{v_{q0}}{s} \\
 \Psi_q(s) = l_q(s) I_q(s) &= \frac{-\omega_b^2 v_{q0} \omega_r^2}{\left(s^2 + \frac{2s}{T_a} + \omega_b^2 \omega_r^2 \right)}
 \end{aligned} \quad (5.121)$$

With $T_a \gg 1/\omega_b$, the total flux linkage components are approximately as follows:

$$\begin{aligned}
 \Psi_d(t) &= \Psi_{d0} + \Psi_d(t) \approx v_{q0} \times e^{-t/T_a} \times \cos \omega_b t \\
 \Psi_q(t) &= 0 + \Psi_q(t) \approx -v_{q0} \times e^{-t/T_a} \times \sin \omega_b t
 \end{aligned} \quad (5.122)$$

Note that due to various approximations, the final flux linkage in axes d and q are zero. In reality (with $r_s \neq 0$), none of them is quite zero.

The electromagnetic torque t_e (P.U.) is

$$t_e(t) = -(\Psi_d(t) I_q(t) - \Psi_q(t) I_d(t)) \quad (5.123)$$

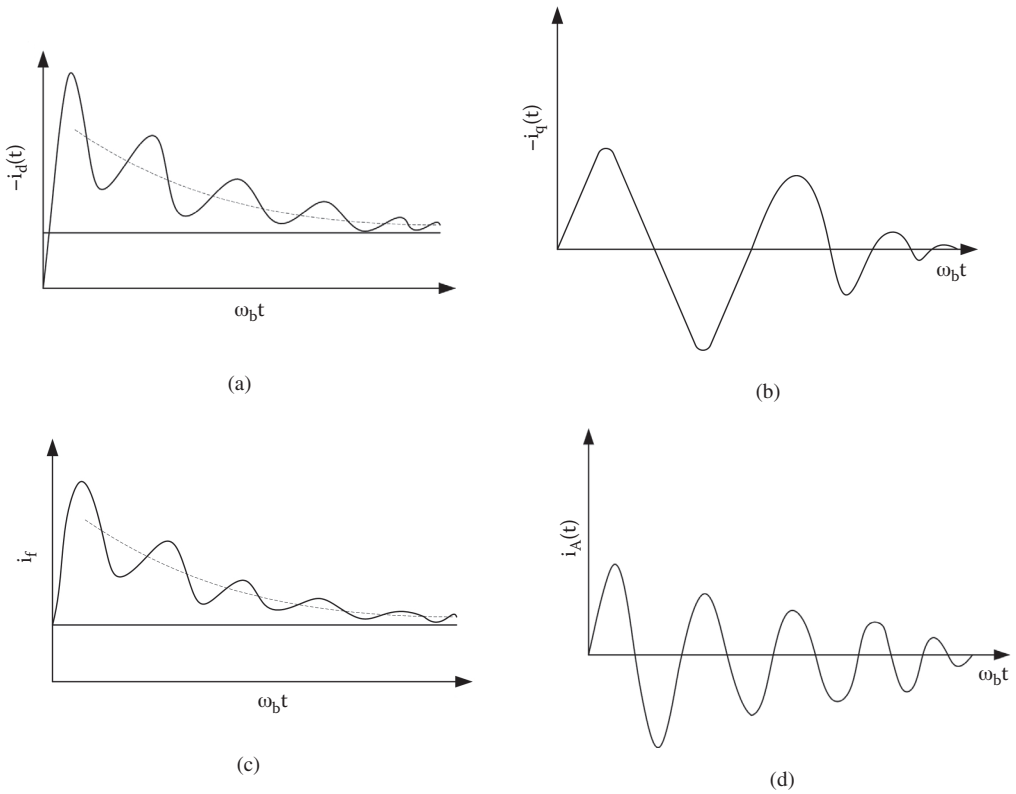


FIGURE 5.11 Sudden short-circuit currents: (a) $I_d(t)$, (b) $I_q(t)$, (c) $I_f(t)$, and (d) $I_A(t)$.

The above approximations ($r_a^2 \approx 0$, $T_a \gg 1/\omega_b$, $\omega_b = \text{ct}$. $T_d', T_{d0}' \gg T_d'', T_{d0}'', T_a$, $r_a < s l_q(s)/\omega_b$) were proven to yield correct results in stator current waveform during unsaturated short-circuit — with given unsaturated values of inductance and time constant terms — within an error below 10%. It may be argued that this is a notable error, but at the same time, we should notice that instrumentation errors are within this range.

The use of Equation 5.118 and Equation 5.120 to estimate the various inductances and time constants, based on measured stator and field current transients during a provoked short-circuit at lower than rated no-load voltage, is included in the standards of both the American National Standards Institute (ANSI) and International Electrotechnical Commission (IEC). Traditionally, grapho-analytical methods of curve fitting were used to estimate the parameters from the sudden three-phase short-circuit. With today's available computing power, various nonlinear programming approaches to SG parameter estimation from short-circuit current versus time curve were proposed [8, 9].

Despite notable progress along this path, there are still uncertainties and notable errors, as both leakage and main flux path magnetic saturation are present and vary during short-circuit transients. In many cases, the speed also varies during short-circuit, while the model assumes it to be constant. To avoid the zero-sequence currents due to nonsimultaneous phase short-circuit, an ungrounded three-phase short-circuit should be performed. Moreover, additional damper cage circuits are to be added for solid rotor pole SGs (turbogenerators) to account for frequency (skin) effects. Sub-subtransient circuits and parameters are introduced to model these effects.

For most large SGs, the sudden three-phase short-circuit test, be it at lower no-load voltage and speed, may be performed only at the user's site, during commissioning, using the turbine as the controlled speed prime mover. This way, the speed during the short-circuit can be kept constant.

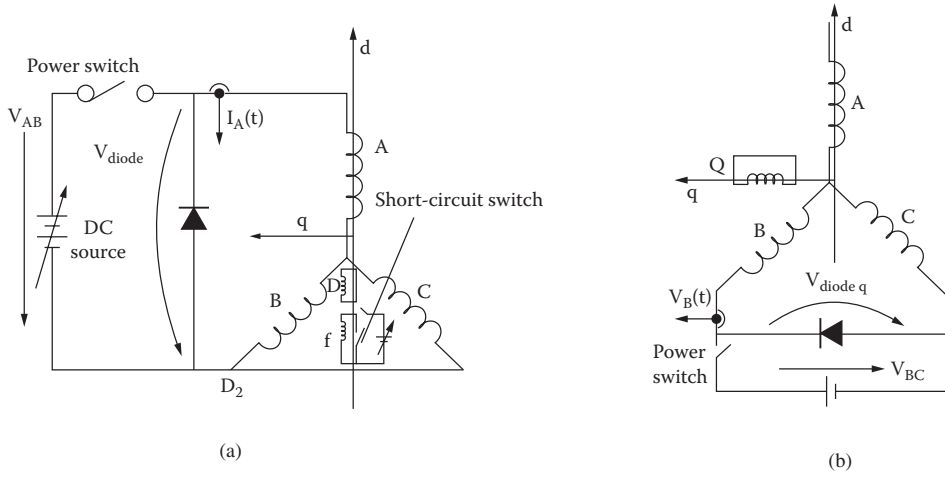


FIGURE 5.12 Arrangement for standstill voltage response transients: (a) axis d and (b) axis q .

5.11 Standstill Time Domain Response Provoked Transients

Flux (current) raise or decay tests may be performed at standstill, with the rotor aligned to axes d and q or for any given rotor position, in order to extract, by curve fitting, the stator current and field current time response for the appropriate SG model. Any voltage-versus-time signal may be applied, but the frequency response standstill tests have recently become accepted worldwide. All of these standstill tests are purely electromagnetic tests, as the speed is kept constant (zero in this case).

The situation in Figure 5.12a corresponds to axis d , while Figure 5.12b refers to axis q .

For axis d ,

$$I_A + I_B + I_C = 0; V_B = V_C; V_A + V_B + V_C = 0$$

$$I_d(t) = \frac{2}{3} \left[I_A + I_B \cos \frac{2\pi}{3} + I_C \cos \left(-\frac{2\pi}{3} \right) \right] = I_A(t)$$

$$I_q(t) = 0$$

$$V_d(t) = \frac{2}{3} \left[V_A + V_B \cos \frac{2\pi}{3} + V_C \cos \left(-\frac{2\pi}{3} \right) \right] = V_A(t) \quad (5.124)$$

$$V_q(t) = 0$$

$$V_A - V_B = V_{diode} = V_A - \left(-\frac{V_A}{2} \right) = \frac{3}{2} V_A = \frac{3}{2} V_d(t)$$

$$V_{ABC} / I_A = \frac{3}{2} V_d / I_d$$

For axis q ,

$$I_A = 0, I_B = -I_C$$

$$V_q = -\frac{2}{3} \left[V_A \sin(0) + V_B \sin \frac{2\pi}{3} + V_C \sin \left(-\frac{2\pi}{3} \right) \right] = -\frac{2}{3} (V_B - V_C) \cdot \frac{\sqrt{3}}{2} = -(V_B - V_C) / \sqrt{3} \quad (5.125)$$

$$I_q = -\frac{2}{3} \left[I_A \sin(0) + I_B \sin \frac{2\pi}{3} + I_C \sin \left(-\frac{2\pi}{3} \right) \right] = -\frac{2}{3} I_B \sqrt{3} = -2 I_B / \sqrt{3} \quad (5.125 \text{ cont.})$$

$$V_q / I_q = (V_B - V_C) / 2 I_B = \frac{V_{BC}}{2 I_B}; I_d = 0$$

Now, for axis d , we simply apply Equation 5.101, with $V_f(s) = 0$, if the field winding is short-circuited, and with $I_q = 0$, $\omega_{r0} = 0$. Also,

$$\begin{aligned} V_d &= +\frac{2}{3} V_{ABC} \\ \frac{2}{3} V_{ABC} \cdot \frac{\omega_b}{s} &= \left(r_s + \frac{s}{\omega_b} l_d(s) \right) I_d(s); I_d = I_A \\ I_f(s) &= -\frac{s}{\omega_b} g(s) I_d(s) \end{aligned} \quad (5.126)$$

For axis q , $I_d = 0$, $\omega_{r0} = 0$:

$$\begin{aligned} V_q &= -\frac{V_{BC}}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} V_{BC} \cdot \frac{\omega_b}{s} &= \left(r_s + \frac{s}{\omega_b} l_q(s) \right) I_q(s); \\ I_q &= 2 I_B / \sqrt{3} \end{aligned} \quad (5.127)$$

The standstill time-domain transients may be explored by investigating both the current rise for step voltage application or current decay when the stator is short-circuited through the freewheeling diode, after the stator was disconnected from the power source.

For current decay, the left-hand side of Equation 5.126 and Equation 5.127 should express only $-2/3 V_{diode}$ in axis d and $\sqrt{3} V_{diode}$ in axis q . The diode voltage $V_{diode}(t)$ should be acquired through proper instrumentation.

For the standard equivalent circuits with $l_d(s)$, $l_q(s)$ having Equation 5.96, from Equation 5.126 and Equation 5.127,

$$\begin{aligned} I_d(s) &= \frac{+\frac{2}{3} V_{AB0}(s) \omega_b}{s \left[r_s + \frac{s}{\omega_b} l_d \frac{(1+sT_d')(1+sT_d'')}{(1+sT_{d0}')(1+sT_{d0}'')} \right]} = I_A(s) \\ I_f(s) &= -\frac{s}{\omega_b} \frac{l_{dm}}{r_f} \frac{1+sT_D}{(1+sT_{d0}')(1+sT_{d0}'')} \cdot I_d(s) \\ I_q(s) &= \frac{-\frac{V_{BC}(s)}{\sqrt{3}} \omega_b}{s \left[r_s + \frac{s}{\omega_b} l_q \frac{(1+sT_q'')}{(1+sT_{q0}'')} \right]} = \frac{2 I_B(s)}{\sqrt{3}} \end{aligned} \quad (5.128)$$

With approximations similar to the case of sudden short-circuit transients, expressions of $I_d(t)$ and $I_f(t)$ are obtained. They are simpler, as no interference from axis q occurs.

In a more general case, where additional damper circuits are included to account for skin effects in solid-rotor SG, the identification process of parameters from step voltage responses becomes more involved. Nonlinear programming methods such as the least squared error, maximum likelihood, and the more recent evolutionary methods such as genetic algorithms could be used to identify separately the d and q inductances and time constants from standstill time domain responses.

The starting point of all such “curve-fitting” methods is the fact that the stator current response contains a constant component and a few aperiodic components with time constant close to T_d'' , T_d' , T_{ad} :

$$\frac{1}{T_{ad}} \approx \frac{r_s}{l_d''} \omega_b \quad (5.129)$$

for axis d , and T_q'' and T_{aq} for axis q :

$$\frac{1}{T_{aq}} \approx \frac{r_s \omega_b}{l_q''} \quad (5.130)$$

When an additional damper circuit is added in axis d , a new time constant T_d''' occurs. Transient and sub-subtransient time constants in axis q (T_q''' , T_q') appear when three circuits are considered in axis q :

$$\begin{aligned} I_d(t) &= I_{d0} + I_d' e^{-t/T_d'} + I_d'' e^{-t/T_d''} + I_{da} e^{-t/T_{da}} + I_d''' e^{-t/T_d'''} \\ I_q(t) &= I_{q0} + I_q' e^{-t/T_q'} + I_q'' e^{-t/T_q''} + I_{qa} e^{-t/T_{aq}} + I_q''' e^{-t/T_q'''} \\ I_f(t) &= I_{f0} + I_f' e^{-t/T_d'} + I_f'' e^{-t/T_d''} + I_f''' e^{-t/T_d'''} + I_{f0} e^{-t/T_{ad}} \end{aligned} \quad (5.131)$$

Curve fitting the measured $I_d(t)$ and $I_q(t)$, respectively, with the calculated ones based on Equation 5.131 yields the time constants and l_d''' , l_d'' , l_d' , l_q''' , l_q'' , l_q' . The main problem with step voltage standstill tests is that they do not properly excite all “frequencies” — time constants — of the machine.

A random cyclic pulse-width modulator (PWM) voltage excitation at standstill seems to be better for parameter identification [11]. Also, for coherency, care must be exercised to set initial (unique) values for stator leakage inductance l_{sl} and for the rotor-to-stator reduction ratio K_f :

$$K_f I_f' = I_f \quad (5.132)$$

In reality, $I_f'(t)$ is acquired and processed, not $I_f(t)$. Finally, it is more often suggested, for practicality, to excite (with a random cyclic PWM voltage) the field winding with short-circuited and opened stator windings rather than the stator, at standstill, because higher saturation levels (up to 25%) may be obtained without overheating the machine. Evidently, such tests are feasible only on axis d (Figure 5.13).

Again, $I_d = I_A$, $V_d = V_q = 0$, and $I_q = 0$. So, from Equation 5.101, with $I_q = 0$ and $V_d(s) = 0$, $V_q(s) = 0$:

$$I_d(s) = \frac{-g(s) \frac{s}{\omega_b} V_f(s)}{r_s + \frac{s}{\omega_b} l_d(s)} \quad (5.133)$$

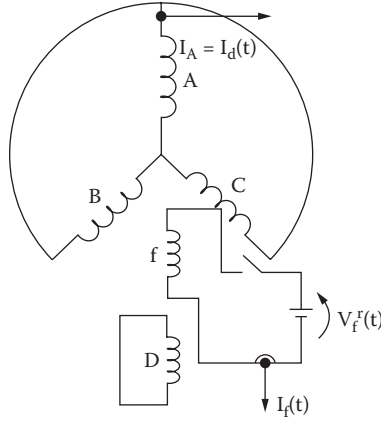


FIGURE 5.13 Field-winding standstill time response test (SSTR) arrangement.

$$I_f(s) = - \frac{v_f(s) \left[1 - g(s) \frac{s^2}{\omega_b^2} \left(1 - \frac{l_{sl}}{r_s + sl_d(s)/\omega_b} \right) \right]}{r_f + \frac{s}{\omega_b} l_{fl}} \quad (5.133 \text{ cont.})$$

Again, the voltage and current rotor/stator reduction ratios, which are rather constant, are needed:

$$I_f = K_f I_f^r; V_f = \frac{2}{3K_f} V_f^r; r_f = \frac{2}{3K_f^2} r_f^r \quad (5.134)$$

To eliminate hysteresis effects (with $V_f(t)$ made from constant height pulses with randomly large timings and zeros) and reach pertinent frequencies, $V_f(t)$ may change polarity cyclically.

The main advantage of standstill time response (SSTR) tests is that the testing time is short.

5.12 Standstill Frequency Response

Another provoked electromagnetic phenomenon at zero speed that is being used to identify SG parameters (inductances and time constants) is the standstill frequency response (SSFR). The SG is supplied in axes d and q , respectively, through a single-phase AC voltage applied to the stator (with the field winding short-circuited) or to the field winding (with the stator short-circuited). The frequency of the applied voltage is varied, in general, from 0.001 Hz to more than 100 Hz, while the voltage is adapted to keep the AC current small enough (below 5% of rated value) to avoid winding overheating. The whole process of raising the frequency level may be mechanized, but considerable testing time is still required. The arrangement is identical to that shown in Figure 5.12 and Figure 5.13, but now $s = j\omega$, with ω in rad/sec.

Equation 5.128 becomes

$$\frac{V_{ABC}}{\underline{I}_A} = \frac{3}{2} \left[r_s + j \frac{\omega}{\omega_b} l_d(j\omega) \right] = \frac{3}{2} Z_d(j\omega) \quad (5.135)$$

$$\underline{I}_f(j\omega) = -j \frac{\omega}{\omega_b} g(j\omega) \cdot \underline{I}_d(j\omega)$$

for axis d , and

$$\frac{V_{BC}}{\underline{I}_B} = 2 \left[r_s + j \frac{\omega}{\omega_b} l_q(j\omega) \right] = 2 \underline{Z}_q(j\omega)$$

for axis q .

Complex number definitions can be used, as a single frequency voltage is applied at any time. The frequency range is large enough to encompass the whole spectrum of electrical time constants that spreads from a few milliseconds to a few seconds.

When the SSFR tests are performed on the field winding (with the stator short-circuited — [Figure 5.13](#)), the response in $\underline{I}_A$ and $\underline{I}_f$ is adapted from Equation 5.133 with $s = j\omega$:

$$\begin{aligned} \underline{I}_d(j\omega) = \underline{I}_A(j\omega) &= \frac{-g(j\omega) j \frac{\omega}{\omega_b} V_f(j\omega)}{r_s + j \frac{\omega}{\omega_b} l_d(j\omega)} \\ \underline{I}_f(j\omega) &= -v_f(j\omega) \frac{\left[1 + g(j\omega) \frac{\omega^2}{\omega_b^2} \left(1 - \frac{l_{sl}}{(r_s + j\omega l_d(j\omega)/\omega_b)} \right) \right]}{rf + j \frac{\omega}{\omega_b} l_{sl}} \end{aligned} \quad (5.136)$$

The general equivalent circuits emanate $l_d(j\omega)$, $l_q(j\omega)$, and $g(j\omega)$. For the standard case, equivalent circuits of [Figure 5.6](#) and Equation 5.96 and Equation 5.97 are used.

For a better representation of frequency (skin) effects, more damper circuits are added along axis d (one, in general; [Figure 5.14a](#)) and along axis q (two, in general; [Figure 5.14b](#)).

The leakage coupling inductance l_{fdl} between the field winding and the damper windings, called also Canay's inductance [14], though generally small (less than stator leakage inductance), proved to be necessary to simultaneously fit the stator current and the field current frequency responses on axis d . Adding even one more such a leakage coupling inductance (say between the two damper circuits on the d axis) failed, so far, to produce improved results but hampered the convergence of the nonlinear programming estimation method used to identify the SG parameters [15].

The main argument in favor of $l_{fdl} \neq 0$ should be its real physical meaning ([Figure 5.15a](#) and [Figure 5.15b](#)).

A myriad of mathematical methods were recently proposed to identify the SG parameters from SSFR, with mean squared error [16] and maximum likelihood [17] being some of the most frequently used. A detailed description of such methods is presented in [Chapter 8](#), dedicated to the testing of SGs.

5.13 Asynchronous Running

When the speed $\omega_r \neq \omega_{r0} = \omega_1$, the stator mmf induces currents in the rotor windings, mainly at slip frequency:

$$S = \frac{\omega_1 - \omega_r}{\omega_1} \quad (5.137)$$

with S the slip.

These currents interact with the stator field to produce an asynchronous torque t_{as} , as in an induction machine. For $S > 0$, the torque is motoring; while for $S < 0$, it is generating. As the rotor magnetic and

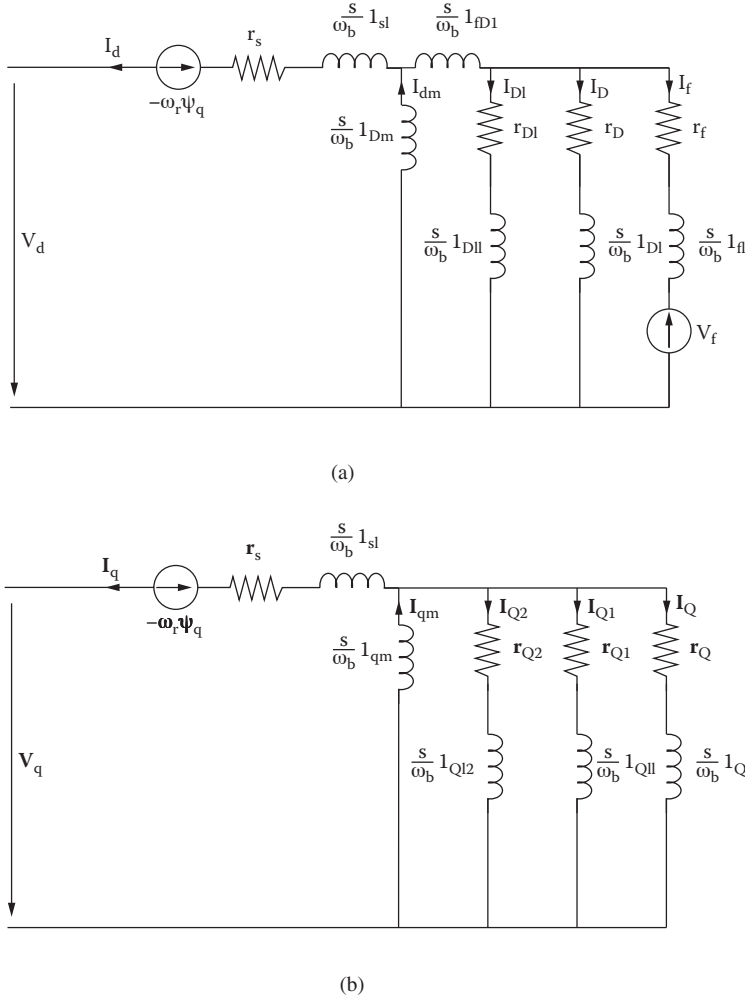


FIGURE 5.14 Synchronous generator equivalent circuits with three rotor circuits: (a) axis d and (b) axis q .

electric circuits are not fully symmetric, there will also be asynchronous torque pulsations, even with $\omega_r \neq \omega_1 = \text{const}$. Also, the average synchronous torque t_{av} is zero as long as $\omega_r \neq \omega_1$.

The d - q model can be used directly to handle transients at a speed $\omega_r \neq \omega_1$. Here we use the d - q model to calculate the average asynchronous torque and currents, considering that the power source that supplies the field winding has a zero internal impedance. That is, the field winding is short-circuited for asynchronous (AC) currents.

The Park transform may be applied to the symmetrical stator voltages:

$$V_{A,B,C}(t) = V \cos \left(\omega_1 \omega_b t - (i-1) \frac{2\pi}{3} \right); [P.U.]$$

$$V_d = -V \cos(\omega_1 \omega_b t - \theta_e)$$

$$V_q = +V \sin(\omega_1 \omega_b t - \theta_e)$$
(5.138)

with

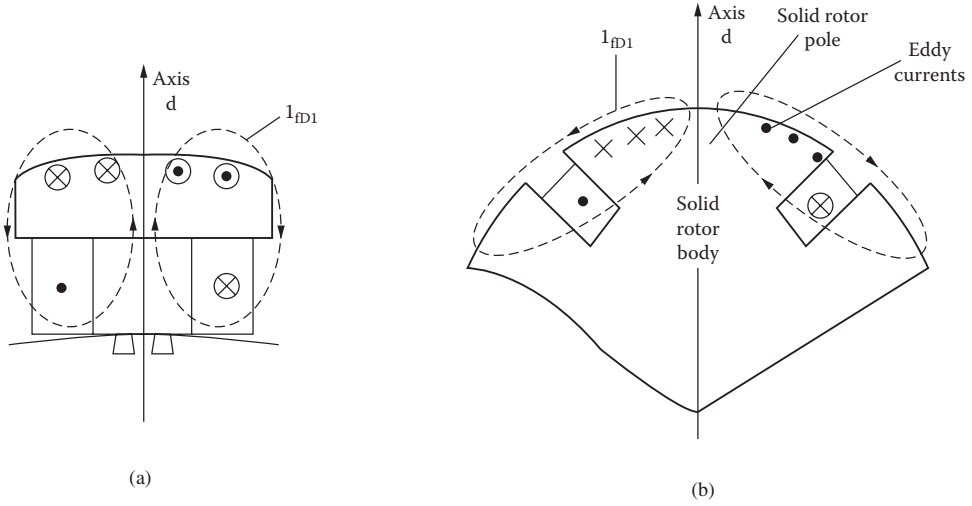


FIGURE 5.15 The leakage coupling inductance l_{fd1} : (a) salient-pole and (b) nonsalient-pole solid rotor.

$$\frac{1}{\omega_b} \frac{d\theta_e}{dt} = \omega_r = ct.$$

$$\theta_e = \int \omega_r \omega_b dt + \theta_0$$

with ω_1 in P.U., ω_b in rad/sec, t in seconds, and V in P.U.

We will now introduce complex number symbols:

$$\begin{aligned} \underline{V}_d &= -V \\ \underline{V}_q &= jV \end{aligned} \quad (5.139)$$

As the speed is constant, d/dt in the d - q model is replaced by the following:

$$\begin{aligned} \frac{1}{\omega_b} \frac{d}{dt} &\rightarrow j(\omega_1 - \omega_r) = jS\omega_1 \\ \omega_r &= \omega_1(1 - S) \end{aligned} \quad (5.140)$$

As the field-winding circuit is short-circuited for the AC current, $\underline{v}_f(jS\omega_1) = 0$.

We will once more use Equation 5.101 in complex numbers and $\omega_r = \omega_1(1 - S)$:

$$\begin{bmatrix} -\underline{V}_d(jS\omega_1\omega_b) \\ -\underline{V}_q(jS\omega_1\omega_b) \end{bmatrix} = \begin{bmatrix} r_s + jS\omega_1 l_d(jS\omega_1\omega_b) & -\omega_1(1-S)l_q(jS\omega_1\omega_b) \\ \omega_1(1-S)l_d(jS\omega_1\omega_b) & r_s + jS\omega_1 l_q(jS\omega_1\omega_b) \end{bmatrix} \begin{bmatrix} \underline{I}_d \\ \underline{I}_q \end{bmatrix} \quad (5.141)$$

Equation 5.141 may be solved for $\underline{I}_d$ and $\underline{I}_q$ with $\underline{V}_d$ and $\underline{V}_q$ from Equation 5.139. The average torque t_{asav} is

$$t_{asav} = -\text{Re} \left[\underline{\Psi}_d(jS\omega_1\omega_b) \underline{I}_q^*(jS\omega_1\omega_b) - \underline{\Psi}_q(jS\omega_1\omega_b) \underline{I}_d^*(jS\omega_1\omega_b) \right] \quad (5.142)$$

with

$$\begin{aligned}\underline{\Psi}_d(jS\omega_1\omega_b) &= \underline{I}_d l_d(jS\omega_1\omega_b); \\ \underline{\Psi}_q(jS\omega_1\omega_b) &= \underline{I}_q l_q(jS\omega_1\omega_b)\end{aligned}\quad (5.143)$$

The field-winding AC current I_f is obtained from Equation 5.135:

$$\underline{I}_f(jS\omega_1\omega_b) = -jS\omega_1 g(jS\omega_1\omega_b) \cdot \underline{I}_d(jS\omega_1\omega_b) \quad (5.144)$$

If the stator resistance is neglected in Equation 5.141,

$$\begin{aligned}\underline{I}_d &= \frac{+V}{j\omega_1 l_d(jS\omega_1\omega_b)} \\ \underline{I}_q &= \frac{-jV}{j\omega_1 l_q(jS\omega_1\omega_b)}\end{aligned}\quad (5.145)$$

In such conditions, the average asynchronous torque is

$$t_{asav} \approx -\frac{v^2}{\omega_1^2} \operatorname{Re} \left[\frac{1}{j l_d^*(jS\omega_1\omega_b)} + \frac{1}{j l_q^*(jS\omega_1\omega_b)} \right], [P.U.] \quad (5.146)$$

The torque is positive when generating (opposite the direction of motion). This happens only for $S < 0$ ($\omega_r > \omega_1$). There is a pulsation in the asynchronous torque due to magnetic anisotropy and rotor circuit unsymmetry. Its frequency is $(2S\omega_1\omega_b)$ in rad/sec. The torque pulsations may be evidenced by switching back from $\underline{I}_d, \underline{I}_q, \underline{\Psi}_d, \underline{\Psi}_q$, complex number form to their instantaneous values:

$$\begin{aligned}I_d(t) &= \operatorname{Re} \left(\underline{I}_d e^{j[(S\omega_1\omega_b)t - \theta_0]} \right) \\ I_q(t) &= \operatorname{Re} \left(\underline{I}_q e^{j[(S\omega_1\omega_b)t - \theta_0]} \right) \\ \Psi_d(t) &= \operatorname{Re} \left(\underline{\Psi}_d e^{j[(S\omega_1\omega_b)t - \theta_0]} \right) \\ \Psi_q(t) &= \operatorname{Re} \left(\underline{\Psi}_q e^{j[(S\omega_1\omega_b)t - \theta_0]} \right) \\ I_A(t) &= I_d(t) \cos(\omega_1\omega_b(1-S)t + \theta_0) - I_q(t) \sin(\omega_1\omega_b(1-S)t + \theta_0)\end{aligned}\quad (5.147)$$

$I_d(t), I_q(t), \Psi_d(t), \Psi_q(t)$ will exhibit components solely at slip frequency, while I_A will show the fundamental frequency ω_1 (P.U.) and the $\omega_1(1-2S)$ component when $r_s \neq 0$.

The instantaneous torque at constant speed in asynchronous running is

$$t_{as}(t) = -(\Psi_d(t)I_q(t) - \Psi_q(t)I_d(t)); [P.U.] \quad (5.148)$$

The $2S\omega_1$ P.U. component (pulsation) in t_{as} is thus physically evident from Equation 5.148. This pulsation may run as high as 50% in P.U. The average torque t_{asav} (P.U.) for an SG with the data $V = 1$,

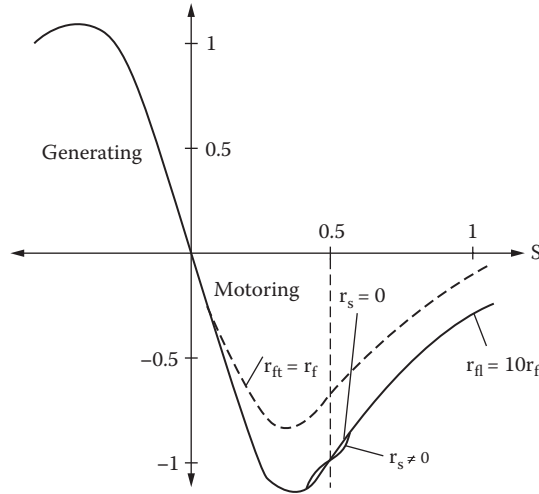


FIGURE 5.16 Asynchronous running of a synchronous generator.

$l_{sl} = 0.15$, $l_{dm} = 1.0$, $l_{fl} = 0.3$, $l_{Dl} = 0.2$, $l_{qm} = 0.6$, $l_{Ql} = 0.12$, $r_s = 0.012$, $r_D = 0.03$, $r_Q = 0.04$, $r_f = 0.03$, and $\underline{V}_f = 0$ is shown in Figure 5.16.

A few remarks are in order:

- The average asynchronous torque may equal or surpass the base torque. As the currents are very large, the machine should not be allowed to work asynchronously for more than 2 min, in general, in order to avoid severe overheating. Also, the SG draws reactive power from the power system while it delivers active power.
- The torque shows a small inflexion around $S = 1/2$ when $r_s \neq 0$. This is not present for $r_s \approx 0$.
- With no additional resistance in the field winding ($r_{ft} = r_f$), no inflexion in the torque–speed curve around $S = 1/2$ occurs. The average torque is smaller in comparison with the case of $r_{ft} = 10r_f$ (additional resistance in the field winding is included).

Example 5.3: Asynchronous Torque Pulsations

For the above data, but with $r_s = 0$ and $\omega_1 = 1$ (rated frequency), derive the formula of instantaneous asynchronous torque.

Solution

From Equation 5.145,

$$\begin{aligned} \underline{I}_d &= \frac{V}{j\bar{l}_d(jS\omega_b)} \\ \underline{I}_q &= \frac{-jV}{j\bar{l}_q(jS\omega_b)} \\ \underline{\Psi}_d &= \bar{l}_d \underline{I}_d = -jV \\ \underline{\Psi}_q &= \bar{l}_q \underline{I}_q = -V \end{aligned} \quad (5.149)$$

Also, let us define $\bar{l}_d(jS\omega_b)$ and $\bar{l}_q(jS\omega_b)$ as follows:

$$\begin{aligned}
 jI_d(jS\omega_b) &= r_d + jI_{dr} = |I_d|e^{j\phi_d} \\
 jI_q(jS\omega_b) &= r_q + jI_{qr} = |I_q|e^{j\phi_q}
 \end{aligned}
 \tag{5.150}$$

According to Equation 5.147, with $\theta_0 = 0$ (for simplicity), from Equation 5.149,

$$\begin{aligned}
 I_d &= \frac{V}{|I_d|} \cos(S\omega_b t - \phi_d) \\
 I_q &= +\frac{V}{|I_q|} \sin(S\omega_b t - \phi_q)
 \end{aligned}
 \tag{5.151}$$

$$\Psi_d = V \sin(S\omega_b t)$$

$$\Psi_q = -V \cos(S\omega_b t)$$

$$\begin{aligned}
 t_{as}(t) &= -(\Psi_d I_q - \Psi_q I_d) = \left[-\frac{V^2}{2} \left(\frac{\cos \phi_d}{|I_d|} + \frac{\cos \phi_q}{|I_q|} \right) + \frac{V^2}{2|I_q|} \cos(2S\omega_b t - \phi_q) - \right. \\
 &\quad \left. - \frac{V^2}{2|I_d|} \cos(2S\omega_b t - \phi_d) \right]
 \end{aligned}
 \tag{5.152}$$

The first term in Equation 5.152 represents the average torque, while the last two refer to the pulsating torque. As expected, for a completely symmetric rotor (as that of an induction machine) the pulsating asynchronous torque is zero, because $|I_d| = |I_q|$, $|\phi_d| = |\phi_q|$ for all slip (speed values).

For motoring ($S > 0$), $\phi_d, \phi_q < 90^\circ$ and for generating ($S < 0$), $90^\circ < \phi_d, \phi_q < 180^\circ$. For small values of slip, the asynchronous torque versus slip may be approximated to a straight line (see [Figure 5.16](#)):

$$t_{asav} \approx -K_{as} S\omega_1 = +K_{as} (\omega_r - \omega_1) \tag{5.153}$$

Equation 5.152 may provide a good basis from which to calculate K_{as} for high-power SGs ($r_s \approx 0$).

Example 5.4: DC Field Current Produced Asynchronous Stator Losses

The DC current in the field winding, if any, produces additional losses in the stator windings through currents at a frequency equal to speed $\omega' = \omega_1(1-S)$ (P.U.). Calculate these losses and their torque.

Solution

In rotor coordinates, these stator d - q currents I'_d, I'_q are DC, at constant speed. To calculate them separately, only the motion-induced voltages are considered, with $V'_d = V'_q = 0$ (the stator is short-circuited, a sign that the power system has a zero internal impedance).

Also, $I'_D = I'_Q = 0$, $I_f = I_{f0}$:

$$\begin{aligned}
 0 &= -(1-S)\omega_1 I'_q + r_s I'_d \\
 0 &= +(1-S)\omega_1 (I'_d + I_{dm} I_{f0}) + r_s I'_q
 \end{aligned}
 \tag{5.154}$$

From Equation 5.154 and for $\omega_1 = 1$,

$$\begin{aligned} I_d' &= \frac{-l_{dm} I_{F0} (1-S)^2 l_q}{r_s^2 + (1-S)^2 l_d l_q} \\ I_q' &= \frac{-l_{dm} I_{F0} (1-S) r_s}{r_s^2 + (1-S)^2 l_d l_q} \end{aligned} \quad (5.155)$$

The stator losses P'_{CO} are as follows:

$$P'_{CO} = \frac{3}{2} r_s (I_d'^2 + I_q'^2) \quad (5.156)$$

The corresponding braking torque t_{as}' is

$$t_{as}' = \frac{P'_{CO}}{\omega_1 (1-S)} > 0 \quad (5.157)$$

The maximum value of t_{as}' occurs at a rather large slip s_K' :

$$S_K' \approx 1 - \sqrt{\frac{2l_d l_q - r_s^2}{2l_q^2 + l_d l_q}} \quad (5.158)$$

The maximum torque t_{as}^{*k} is as follows:

$$t_{as}^{*k} \approx t_{as}'(S_K') \quad (5.159)$$

Close to the synchronous speed ($S = 0$), this torque becomes negligible.

5.14 Simplified Models for Power System Studies

The P.U. system of SG equations (Equation 5.67) describes completely the standard machine for any transients. The complexity of such a model makes it less practical for power system stability studies, where tens or hundreds of SGs and consumers are involved and have to be modeled. Simplifications in the SG model are required for such a purpose. Some of them are discussed below, while more information is available in the literature on power system stability and control [1, 17].

5.14.1 Neglecting the Stator Flux Transients

When neglecting the stator transients in the d - q model, it means to make $\frac{\partial \Psi_d}{\partial t} = \frac{\partial \Psi_q}{\partial t} = 0$. It was demonstrated that it is also necessary to simultaneously consider — only in the stator voltage equations — constant (synchronous) speed:

$$\begin{aligned}
V_d &= -I_d r_s + \Psi_q \omega_{r0} \\
V_q &= -I_q r_s - \Psi_d \omega_{r0} \\
\frac{1}{\omega_b} \frac{d\Psi_f}{dt} &= -I_f r_f + V_f \\
\frac{1}{\omega_b} \frac{d\Psi_D}{dt} &= -I_D r_D \\
\frac{1}{\omega_b} \frac{d\Psi_Q}{dt} &= -I_Q r_Q \\
\Psi_d &= l_{sl} I_d + l_{dm} (I_d + I_f + I_D) \\
\Psi_q &= l_{sl} I_q + l_{qm} (I_q + I_Q) \\
\Psi_f &= l_{fl} I_f + l_{dm} (I_d + I_f + I_D) + l_{fdl} (I_f + I_D) \\
\Psi_D &= l_{fl} I_D + l_{dm} (I_d + I_f + I_D) + l_{fdl} (I_f + I_D) \\
\Psi_Q &= l_{ql} I_q + l_{qm} (I_q + I_Q) \\
2H \frac{d\omega_r}{dt} &= t_{shaft} - t_e; t_e = \Psi_d I_q - \Psi_q I_d; \omega_b \frac{d\delta_v}{dt} = \omega_r - \omega_0 \\
\frac{1}{\omega_b} \frac{d\theta_{er}}{dt} &= \omega_r; \left| \begin{matrix} V_d \\ V_q \\ V_0 \end{matrix} \right| = \left| P(\theta_{er}) \right| \left| \begin{matrix} V_a \\ V_b \\ V_c \end{matrix} \right|
\end{aligned} \tag{5.160}$$

The flux and current relationships are the same as in Equation 5.67. The state variables may be I_d , I_q , Ψ_f , Ψ_D , Ψ_Q , ω_r , and θ_{er} . I_d and I_q are calculated from the, now algebraic, equations of stator. The system order was reduced by two units.

As expected, fast 50 (60) Hz frequency transients, occurring in I_d , I_q , and t_e , are eliminated. Only the “average” transient torque is “visible.” Allowing for constant (synchronous) speed in the stator equations

with $\frac{\partial \Psi_d}{\partial t} = \frac{\partial \Psi_q}{\partial t} = 0$ counteracts the effects of such an approximation, at least for small signal transients, in terms of speed and angle response [17]. By neglecting stator transients, we are led to steady-state stator voltage equations. Consequently, if the power network transients are neglected, the connection of the SG model to the power network model is rather simple, with steady state all over.

A drastic computation time saving is thus obtained in power system stability studies.

5.14.2 Neglecting the Stator Transients and the Rotor Damper Winding Effects

This time, in addition, the damper winding currents are zero, $I_D = I_Q = 0$, and thus,

$$\begin{aligned}
V_d &= -I_d r_s + \Psi_q \omega_{r0} \\
V_q &= -I_q r_s - \Psi_d \omega_{r0}
\end{aligned} \tag{5.161}$$

$$\begin{aligned}
\frac{1}{\omega_b} \frac{d\Psi_f}{dt} &= -I_f r_f + V_f \\
\Psi_d &= l_{sl} I_d + l_{dm} (I_d + I_f) \\
\Psi_q &= l_{sl} I_q + l_{qm} I_q \\
\Psi_f &= l_{\beta} I_f + l_{dm} (I_d + I_f) \\
2H \frac{d\omega_r}{dt} &= t_{shaft} - t_e; t_e = \Psi_d I_q - \Psi_q I_d; \omega_b \frac{d\delta v}{dt} = \omega_r - \omega_{r0} \\
\frac{1}{\omega_b} \frac{d\theta_{er}}{dt} &= \omega_r; \begin{vmatrix} V_d \\ V_q \\ V_0 \end{vmatrix} = \left| P(\theta_{er}) \right| \begin{vmatrix} V_A \\ V_B \\ V_C \end{vmatrix}
\end{aligned} \tag{5.161 cont.}$$

The order of the system was further reduced by two units. An additional computation time saving is obtained, with only one electrical transient left — the one produced by the field winding. The model is adequate for slow transients (seconds and more).

5.14.3 Neglecting All Electrical Transients

The field current is now considered constant. We are dealing with very slow (mechanical) transients:

$$\begin{aligned}
V_d &= -I_d r_s + \Psi_q \omega_{r0} \\
V_q &= -I_q r_s - \Psi_d \omega_{r0} \\
I_f &= V_f / r_f \\
2H \frac{d\omega_r}{dt} &= t_{shaft} - t_e; t_e = \Psi_d I_q - \Psi_q I_d; \omega_b \frac{d\delta v}{dt} = \omega_r - \omega_{r0} \\
\omega_b \frac{d\theta_{er}}{dt} &= \omega_r; \begin{vmatrix} V_d \\ V_q \\ V_0 \end{vmatrix} = \left| P(\theta_{er}) \right| \begin{vmatrix} V_A \\ V_B \\ V_C \end{vmatrix}
\end{aligned} \tag{5.162}$$

This time, we start again with initial values of variables: I_{d0} , I_{q0} , I_{f0} , $\omega_r = \omega_{r0}$, $\delta_{v0}(\theta_{er0})$ and V_{d0} , V_{q0} , $t_{e0} = t_{shaft0}$. So, the machine is under steady state electromagnetically, while making use of the motion equation to handle mechanical transients. In very slow transients (tens of seconds), such a model is appropriate. Note that a plethora of constant flux approximate models (with or without rotor damper cage) in use for power system studies [1, 17] are not followed here [17]. Among the simplified models we illustrate here, only the “mechanical” model is followed, as it helps in explaining SG self-synchronization, step shaft torque response, and SG oscillations (free and forced).

5.15 Mechanical Transients

When the prime-mover torque varies in a nonperiodical or periodical fashion, the large inertia of the SG leads to a rather slow speed (power angle δ_v) response. To evidenciate such a response, the electromagnetic transients may be altogether neglected, as suggested by the “mechanical model” presented in the previous paragraph.

As the speed varies, an asynchronous torque t_{as} occurs, besides the synchronous torque. The motion equation becomes (in P.U.) as follows:

$$2H \frac{d\omega_r}{dt} = t_{shaft} - t_e - t_{as} \quad (5.163)$$

with ($r_s = 0$)

$$t_e = e_f \frac{V \sin \delta_V}{l_d} + \frac{V^2}{2} \left(\frac{1}{l_q} - \frac{1}{l_d} \right) \sin 2\delta_V \quad (5.164)$$

$$t_{as} = \frac{K_{as}}{\omega_b} \frac{d\delta_V}{dt};$$

$$\frac{1}{\omega_b} \frac{d\delta_V}{dt} = \omega_r - \omega_1 \quad (5.165)$$

and $e_f = l_{dm} I_f$ represents the no-load voltage for a given field current.

Only the average asynchronous torque is considered here. The model in Equation 5.163 through Equation 5.165 may be solved numerically for ω_r and δ_V as variables once their initial values are given together with the prime-mover torque t_{shaft} versus time or versus speed, with or without a speed governor. For small deviations, Equation 5.163 through Equation 5.165 become

$$\frac{2H}{\omega_b} \frac{d^2 \Delta \delta_V}{dt^2} + \left(\frac{\partial t_e}{\partial \delta_V} \right)_{\delta_{V0}} \cdot \Delta \delta_V + \frac{K_{as}}{\omega_b} \frac{d \Delta \delta_V}{dt} = \Delta t_{shaft} \quad (5.166)$$

$$(t_e)_{\delta_{V0}} = t_{shaft0}$$

$$\left(\frac{\partial t_e}{\partial \delta_V} \right)_{\delta_{V0}} = t_{es0} \quad (5.167)$$

Equality (Equation 5.167) reflects the starting steady-state conditions at initial power angle δ_{V0} . t_{es0} is the so-called synchronizing torque (as long as $t_{es0} > 0$, static stability is secured).

5.15.1 Response to Step Shaft Torque Input

For step shaft torque input, Equation 5.166 allows for an analytical solution:

$$\Delta \delta_V(t) = \frac{\Delta t_{shaft}}{t_{es0}} + \text{Re} \left[A_1 e^{\gamma_1 t} + B_1 e^{\gamma_2 t} \right] \quad (5.168)$$

$$\gamma_{1,2} = \frac{-\left(K_{as} / \omega_b\right) \pm \sqrt{\left(\frac{K_{as}}{\omega_b}\right)^2 - \frac{8H \cdot t_{es0}}{\omega_b}}}{4\left(H / \omega_b\right)} = -\frac{1}{T_{as}} \pm j\omega' \quad (5.169)$$

$$\begin{aligned}\frac{1}{T_{as}} &= \frac{K_{as}}{4H}; \\ (\omega')^2 &= -\left(\frac{1}{T_{as}}\right)^2 + \omega_0^2; \\ \omega_0 &= \sqrt{\frac{t_{es0}\omega_b}{2H}}\end{aligned}\tag{5.170}$$

Finally,

$$\Delta\delta_V(t) = \frac{\Delta t_{shaft}}{t_{es0}} \left[1 - e^{-\frac{t}{T_{as}}} \frac{\sin(\omega t + \Psi)}{\sin \Psi} \right]$$

The constant Ψ is obtained by assuming initial steady-state conditions:

$$\begin{aligned}(\Delta\delta_V)_{t=0} &= 0, \\ \left(\frac{d(\Delta\delta)}{dt}\right)_{t=0} &= 0\end{aligned}\tag{5.171}$$

Finally,

$$\tan \Psi = \omega' T_{as}\tag{5.172}$$

The power angle and speed response for step shaft torque are shown qualitatively in Figure 5.17.

Note that ω_0 (Equation 5.170) is traditionally known as the proper mechanical frequency of the SG. Unfortunately, ω_0 varies with power angle δ_V , field current I_{f0} , and with inertia. It decreases with increasing δ_V and increases with increasing I_{f0} .

In general, $f_0 = \omega_0/2\pi$ varies from less than or about 1 Hz to a few hertz for large and medium power SGs, respectively.

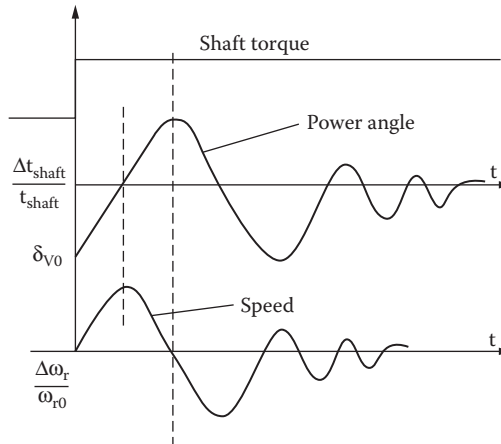


FIGURE 5.17 Power angle and speed responses to step shaft torque input.

5.15.2 Forced Oscillations

Shaft torque oscillations may occur due to various reasons. The diesel engine prime movers are a typical case, as their torque varies with rotor position. The shaft torque oscillations may be written as follows:

$$\Delta t_{shaft} = \sum t_{shv} \cos(\Omega_v t - \Psi_v) \quad (5.173)$$

with Ω_v in rad/sec.

Consider first an autonomous SG without any asynchronous torque ($K_{as} = 0$). This is the ideal case of free oscillations.

From Equation 5.166,

$$\frac{2H}{\omega_b} \frac{d^2 \Delta \delta_v}{dt^2} = \sum t_{shv} \cos(\Omega_v t - \Psi_v) \quad (5.174)$$

The steady-state solution of this equation is straightforward:

$$\begin{aligned} \Delta \delta_{vv}^a &= -\Delta \delta_{vvm} \cos(\Omega_v t - \Psi_v) \\ -\frac{\omega_b t_{shv}}{2H\Omega_v^2} &= \Delta \delta_{vvm}^a \end{aligned} \quad (5.175)$$

The amplitude of this free oscillation (for harmonic v) is thus inversely proportional to inertia and to the frequency of oscillation squared.

For the SG with rotor damper cage and connected to the power system, both K_{as} and t_{es0} (synchronizing torque) are nonzero; thus, Equation 5.166 has to be solved as it is:

$$\frac{2H}{\omega_b} \frac{d^2 \Delta \delta_v}{dt^2} + \frac{K_{as}}{\omega_b} d \frac{\Delta \delta_v}{dt} + t_{es0} \Delta \delta_v = \sum \Delta t_{shv} \cos(\Omega_v t - \Psi_v) \quad (5.176)$$

Again, the steady-state solution is sought:

$$\Delta \delta_{vv} = \Delta \delta_{vvm} \sin(\Omega_v t - \Psi_v - \phi_v) \quad (5.177)$$

with

$$\begin{aligned} \Delta \delta_{vvm} &= \frac{\Delta t_{shv}}{\sqrt{\left(\left(\frac{2H}{\omega_b} \Omega_v^2 - t_{es0} \right)^2 + \left(\frac{K_{as}}{\omega_b} \Omega_v \right)^2}}; \\ \phi_v &= \tan^{-1} \left(\frac{\left(-\frac{2H}{\omega_b} \Omega_v^2 + t_{es0} \right)^{-1}}{K_{as} \frac{\Omega_v}{\omega_b}} \right) \end{aligned} \quad (5.178)$$

The ratio of the power angle amplitudes $\Delta \delta_{vvm}$ and $\Delta \delta_{vvm}^a$ of forced and free oscillations, respectively, is called the modulus of mechanical resonance K_{mv} :

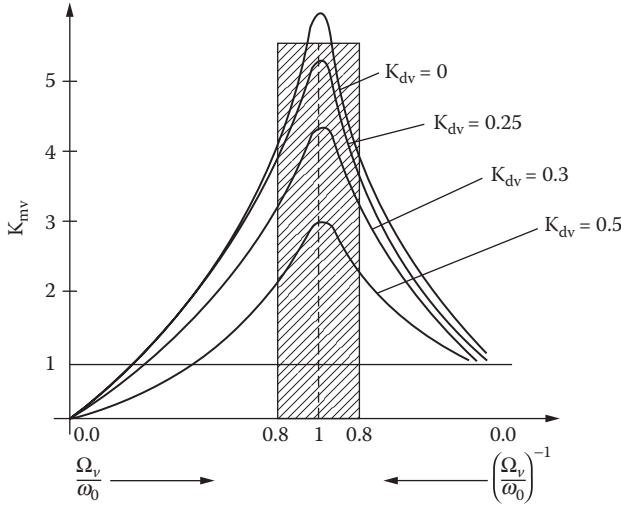


FIGURE 5.18 The modulus of mechanical resonance K_m .

$$K_{mv} = \frac{\Delta\delta_{vvm}}{\Delta\delta_{vvm}^a} = \frac{1}{\sqrt{\left(1 - \frac{\omega_0^2}{\Omega_v^2}\right)^2 + \left(\frac{K_{as}}{2H\Omega_v}\right)^2}} \quad (5.179)$$

The damping coefficient K_{dv} is

$$K_{dv} = \frac{K_{as}}{2H\Omega_v} \quad (5.180)$$

Typical variations of K_{mv} with the ω_0/Ω_v ratio for various K_{dv} values are given in Figure 5.17. The resonance conditions for $\omega_0 = \Omega_v$ are evident. To reduce the amplification effect, K_{dv} is increased, but this is feasible only up to a point by enforcing the damper cage (more copper). So, in general, for all shaft torque frequencies, Ω_v , it is appropriate to fall outside the hatched region in Figure 5.18:

$$1.25 > \frac{\omega_0}{\Omega_v} > 0.8 \quad (5.181)$$

As ω_0 — the proper mechanical frequency (Equation 5.170) — varies with load (δ_v) and with field current for a given machine, the condition in Equation 5.181 is not so easy to fulfill for all shaft torque pulsations. The elasticity of shafts and of mechanical couplings between them in an SG set is a source of additional oscillations to be considered for the constraint in Equation 5.181. The case of the autonomous SG with damper cage rotor requires a separate treatment.

5.16 Small Disturbance Electromechanical Transients

After the investigation of fast (constant speed; electromagnetic) and slow (mechanical) transients, we return to the general case when both electrical and mechanical transients are to be considered.

Electric power load variations typically cause such complex transients.

For multiple SGs and loads, power systems, voltage, and frequency control system design are generally based on small disturbance theories in order to capitalize on the theoretical heritage of linear control systems and reduce digital simulation time.

In essence, the complete (or approximate) d - q model (Equation 5.67) of the SG is linearized about a chosen initial steady-state point by using only the first Taylor's series component. The linearized system is written in the state-space form:

$$\begin{aligned}\dot{\Delta X} &= A\Delta X + B\Delta V \\ \Delta Y &= C\Delta X + D\Delta V\end{aligned}\tag{5.182a}$$

where

ΔX = the state variables vector
 ΔV = the input vector
 ΔY = the output vector

The voltage and speed controller systems may be included in Equation 5.182; thus, the small disturbance stability of the controlled generator is investigated by the eigenvalue method:

$$\det(A - \lambda I) = 0\tag{5.182b}$$

with I being the unity diagonal matrix.

The eigenvalues λ may be real or complex numbers. For system stability despite small disturbances, all eigenvalues should have a negative real part.

The unsaturated d - q model (Equation 5.87) in P.U. may be linearized as follows:

$$\begin{aligned}\Delta V_d &= -\Delta I_d r_s - \frac{1}{\omega_b} d \left(\frac{\Delta \Psi_d}{dt} \right) + \omega_{r0} \Delta \Psi_q + \Delta \omega_r \Psi_{q0} \\ \Delta V_q &= -\Delta I_q r_s - \frac{1}{\omega_b} d \left(\frac{\Delta \Psi_q}{dt} \right) - \omega_{r0} \Delta \Psi_d - \Delta \omega_r \Psi_{d0} \\ \Delta V_f &= \Delta I_f r_f + \frac{1}{\omega_b} d \left(\frac{\Delta \Psi_f}{dt} \right) \\ 0 &= -\Delta I_D r_D - \frac{1}{\omega_b} d \left(\frac{\Delta \Psi_D}{dt} \right) \\ 0 &= -\Delta I_Q r_Q - \frac{1}{\omega_b} d \left(\frac{\Delta \Psi_Q}{dt} \right)\end{aligned}\tag{5.183}$$

$$\begin{aligned}\Delta t_{shaft} &= \Delta t_e + 2H \frac{d}{dt} (\Delta \omega_r) \\ \Delta t_e &= -(\Psi_{d0} \Delta I_q + I_{q0} \Delta \Psi_{d0} - \Psi_{q0} \Delta I_d - I_{d0} \Delta \Psi_q) \\ \frac{1}{\omega_b} \frac{d \Delta \delta_V}{dt} &= \Delta \omega_r\end{aligned}\tag{5.184}$$

For the initial (steady-state) point,

$$\begin{aligned}
 V_{d0} &= -I_{d0}r_s + \Psi_{q0}\omega_{r0} \\
 V_{q0} &= -I_{q0}r_s - \Psi_{d0}\omega_{r0} \\
 \Psi_{d0} &= l_{sl}I_{d0} + l_{dm}(I_{d0} + I_{f0}) \\
 \Psi_{q0} &= (l_{sl} + l_{qm})I_{q0} \\
 V_{f0} &= I_{f0}r_f; I_{D0} = I_{Q0} = 0 \\
 t_{e0} &= -(\Psi_{d0}I_{d0} - \Psi_{q0}I_{q0}) = t_{shaft} \\
 \delta_V &= \tan^{-1} \frac{V_{d0}}{V_{q0}}
 \end{aligned} \tag{5.185}$$

$$\begin{aligned}
 V_{d0} &= -V_0 \sin \delta_0 \\
 V_{q0} &= -V_0 \cos \delta_0 \\
 \Delta V_d &\approx -\Delta V \sin \delta_{V0} - V_0 \cos \delta_{V0} \Delta \delta_V \\
 \Delta V_q &\approx -\Delta V \cos \delta_{V0} + V_0 \sin \delta_{V0} \Delta \delta_V \\
 \Delta \Psi_d &= l_{sl} \Delta I_d + l_{dm} \Delta I_{dm}; \Delta I_{dm} = \Delta I_d + \Delta I_f + \Delta I_D \\
 \Delta \Psi_q &= l_{sl} \Delta I_q + l_{qm} \Delta I_{qm}; \Delta I_{qm} = \Delta I_q + \Delta I_Q \\
 \Delta \Psi_D &= l_{Dl} \Delta I_D + l_{dm} \Delta I_{dm} \\
 \Delta \Psi_Q &= l_{Ql} \Delta I_Q + l_{qm} \Delta I_{qm}
 \end{aligned} \tag{5.186}$$

Eliminating the flux linkage disturbances in Equation 5.183 and Equation 5.184 by using Equation 5.186 leads to the definition of the following state-space variable vector X :

$$|X| = (\Delta I_{ds}, \Delta I_f, \Delta I_{dm}, \Delta I_{qs}, \Delta I_{qm}, \Delta \omega_r, \Delta \delta_V)^t \tag{5.187}$$

The input vector is

$$|\Delta V| = [-\Delta V \sin \delta_0, \Delta V_f, 0, -\Delta V \cos \delta_0, 0, \Delta t_{shaft}, 0]^t \tag{5.188}$$

We may now put Equation 5.183 and Equation 5.184 with Equation 5.186 into matrix form as follows:

$$|\Delta V| = -|L| \frac{1}{\omega_b} d \frac{(\Delta X)}{dt} - |R| \Delta X \tag{5.189}$$

with

$$|L| = \begin{array}{c|ccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 1 & l_{sl} & 0 & l_{dm} & 0 & 0 & 0 & 0 \\ 2 & 0 & l_{fl} & l_{dm} & 0 & 0 & 0 & 0 \\ 3 & -l_{sl} & -l_{dl} & l_{dm} + l_{dl} & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & l_{sl} & l_{qm} & 0 & 0 \\ 5 & 0 & 0 & 0 & -l_{ql} & l_{qm} + l_{ql} & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 7 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{array}$$

$$|R| = \begin{array}{c|ccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 1 & r_s & 0 & 0 & -\omega_{r0} l_{sl} & -\omega_{r0} l_{qm} & -\Psi_{q0} & V_0 \cos \delta_{V0} \\ 2 & 0 & r_f & 0 & 0 & 0 & 0 & 0 \\ 3 & -r_D & -r_D & +r_D & 0 & 0 & 0 & 0 \\ 4 & \omega_{r0} l_{sl} & 0 & \omega_{r0} l_{dm} & r_s & 0 & \Psi_{d0} & -V_0 \sin \delta_{V0} \\ 5 & 0 & 0 & 0 & -r_Q & r_Q & 0 & 0 \\ 6 & \frac{l_{sl} I_{q0} - \Psi_{q0}}{2H\omega_b} & 0 & \frac{l_{dm} I_{q0}}{2H\omega_b} & \frac{\Psi_{d0} - l_{sl} I_{d0}}{2H\omega_b} & \frac{-l_{dm} I_{d0}}{2H\omega_b} & -1 & 0 \\ 7 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array}$$

Comparing Equation 5.182 with Equation 5.189,

$$\begin{aligned} A &= -|L|^{-1} \cdot |R|; \\ B &= -[L]^{-1} |1, 1, 1, 1, 1, 1| \end{aligned} \quad (5.190)$$

All that remains is calculation of the eigenvalues of matrix A, and thus establishing the small disturbance stability performance of a SG. The choice of I_{dm} and I_{qm} as variables, instead of rotor damper cage currents I_d and I_q , makes $[L]$ more sparse and leaves way to somehow consider magnetic saturation, at least for steady state when the dependences of l_{dm} and l_{qm} on both I_{dm} and I_{qm} (Figure 5.10) may be established in advance by tests or by finite element calculations.

This is easy to apply, as values of I_{dm0} and I_{qm0} are straightforward:

$$\begin{aligned} I_{dm0} &= I_{d0} + I_{f0} \\ I_{qm0} &= I_{q0} \end{aligned} \quad (5.191)$$

and the level of saturation may be considered “frozen” at the initial conditions (not influenced by small perturbations). Typical critical eigenvalue changes with active power for the I_s, I_f, I_m model above are shown in Figure 5.19 [18].

It is possible to choose the variable vector in different ways by combining various flux linkages and current as variables [18]. There is not much to gain with such choices, unless magnetic saturation is not rigorously considered. It was shown that care must be exercised in representing magnetic saturation by

single magnetization curves (dependent on $I_m = \sqrt{I_{dm}^2 + I_{qm}^2}$) in the underexcited regimes of SG when large errors may occur. Using only complete $I_{dm}(\Psi_{dm}, \Psi_{qm}), I_{qm}(\Psi_{dm}, \Psi_{qm})$ families of saturation curves will lead to good results throughout the whole active and reactive power capability region of the SG.

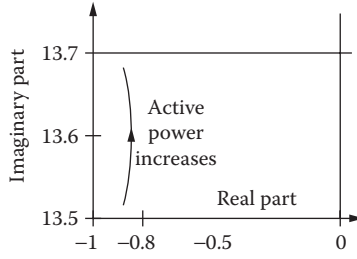


FIGURE 5.19 Critical mode eigenvalues of unsaturated (I_s , I_f , I_m) model when active power rises from 0.8 to 1.2 per unit (P.U.).

As during small perturbation transients the initial steady-state level is paramount, we leave out the transient saturation influence here, to consider it in the study of large disturbance transients when the latter matters notably.

5.17 Large Disturbance Transients Modeling

Large disturbance transients modeling of a single SG should take into account both magnetic saturation and frequency (skin rotor) effects.

To complete a d - q model, with two d axis damper circuits and three q axis damper circuits, satisfies such standards, if magnetic saturation is included, even if included separately along each axis (see Figure 5.20a for the d axis and Figure 5.20b for the q axis). For the sake of completion, all equations of such a model are given in what follows:

$$\begin{aligned}
 \Psi_d &= l_{sl} I_d + \Psi_{dm} \\
 \Psi_q &= l_{sl} I_q + \Psi_{qm} \\
 \Psi_f &= l_{fl} I_f + \Psi_{dm} + l_{fdl} (I_f + I_D + I_{D1}) \\
 \Psi_D &= l_{Dl} I_D + \Psi_{dm} + l_{fdl} (I_f + I_D + I_{D1}) \\
 \Psi_{D1} &= l_{D1l} I_{D1} + \Psi_{dm} + l_{fdl} (I_f + I_D + I_{D1}) \\
 \Psi_Q &= l_{Ql} I_Q + \Psi_{qm} \\
 \Psi_{Q1} &= l_{Q1l} I_{Q1} + \Psi_{qm} \\
 \Psi_{Q2} &= l_{Q2l} I_{Q2} + \Psi_{qm} \\
 I_{dm} &= I_d + I_D + I_{D1} + I_f \\
 I_{qm} &= I_q + I_Q + I_{Q1} + I_{Q2} \\
 \Psi_{dm} &= l_{dm} (I_{dm}, I_{qm}) I_{dm} \\
 \Psi_{qm} &= l_{qm} (I_{dm}, I_{qm}) I_{qm}
 \end{aligned} \tag{5.192}$$

$$\tag{5.193}$$

The magnetization curve families in Equation 5.193 have to be obtained either through standstill time response tests or through FEM magnetostatic calculations at standstill. Once analytical polynomial spline

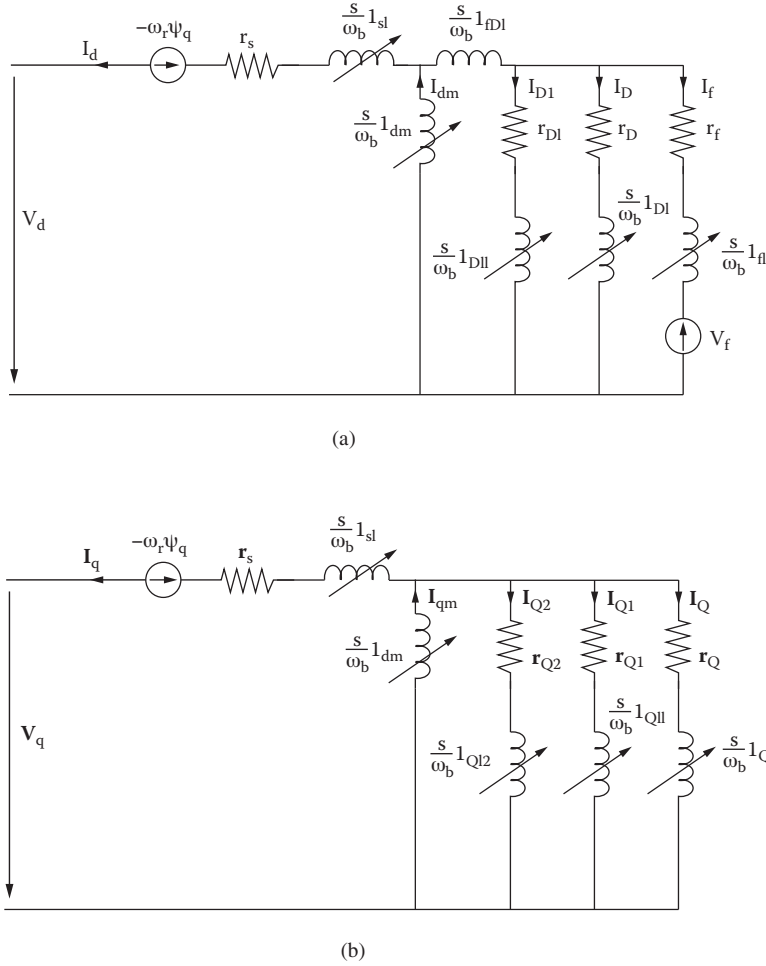


FIGURE 5.20 Three-circuit synchronous generator model with some variable inductances: (a) axis d and (b) axis q .

approximation from l_{dm} and l_{qm} functions are obtained, the time derivatives of Ψ_{dm} and Ψ_{qm} may be obtained as follows:

$$\begin{aligned} \frac{s}{\omega_b} \frac{d\Psi_{dm}}{dt} &= L_{ddm} \frac{s}{\omega_b} \frac{dI_{dm}}{dt} + L_{qdm} \frac{s}{\omega_b} \frac{dI_{qm}}{dt} \\ \frac{s}{\omega_b} \frac{d\Psi_{qm}}{dt} &= L_{dqm} \frac{s}{\omega_b} \frac{dI_{dm}}{dt} + L_{qqm} \frac{s}{\omega_b} \frac{dI_{qm}}{dt} \end{aligned} \quad (5.194)$$

$$l_{ddm} = l_{dm} + \frac{\partial l_{dm}}{\partial I_{dm}} I_{dm}; l_{qdm} = \frac{\partial l_{dm}}{\partial I_{qm}} I_{dm}$$

$$l_{dqm} = \frac{\partial l_{qm}}{\partial I_{dm}} I_{qm}; l_{qqm} = l_{qm} + \frac{\partial l_{qm}}{\partial I_{qm}} I_{qm}$$

The leakage saturation occurs only in the field winding and in the stator winding and is considered to depend solely on the respective currents:

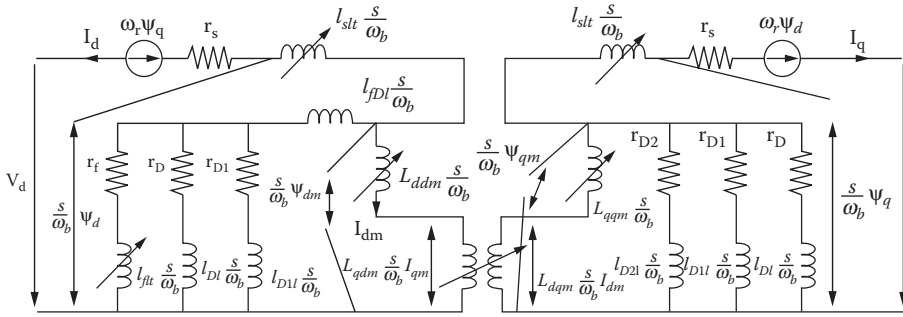


FIGURE 5.21 General three-circuit synchronous generator model with cross-coupling saturation.

$$\frac{s}{\omega_b} l_{sl}(I_s) = \left(l_{sl}(I_s) + \frac{\partial l_{sl}}{\partial I_s} I_s \right) \frac{s}{\omega_b} = l_{sl}(I_s) \frac{s}{\omega_b}$$

$$I(s) = \sqrt{I_d^2 + I_q^2} \quad (5.195)$$

$$\frac{s}{\omega_b} l_{fl}(I_f) = \left(l_{fl}(I_f) + \frac{\partial l_{fl}}{\partial I_f} I_f \right) \frac{s}{\omega_b} = l_{fl}(I_f) \frac{s}{\omega_b}$$

So, the leakage inductances of stator and field winding in the equivalent circuit are replaced by their transient value l_{sl} , l_{fl} . However, in the flux–current relationship, their steady-state values l_{sl} , l_{fl} occur. They are all functions of their respective currents. It should be mentioned that only for currents in excess of 2 P.U., is leakage saturation influence worth considering. A sudden short-circuit represents such a transient process.

Equation 5.194 suggests that a cross-coupling between the equivalent circuits of axes d and q is required. This is simple to operate (Figure 5.21). From the reciprocity condition, $l_{daqm} = l_{qdm}$. And the general equivalent circuit may be identified:

- The magnetization curve family (Equation 5.193 and Equation 5.194) and leakage inductance functions $l_{sl}(I_s)$, $l_{fl}(I_f)$ are first determined from time domain standstill tests (or FEM).
- From frequency response at standstill or through FEM, all the other components are calculated.

The dependence of l_{dm} and l_{qm} on both I_{dm} and I_{qm} should lead to model suitability in all magnetization conditions, including the disputed case of underexcited SG when the concept of $l_{dm}(I_m)$, $l_{qm}(I_m)$ unique functions or of total magnetization current (mmf) fails [4].

The machine equations are straightforward from the equivalent scheme and thus are not repeated here. The choice of variables is as in the paragraph on small perturbations. Tedious FEM tests and their processing are required before such a complete circuit model of the SG is established in all rigor.

That the d – q model may be used to investigate various symmetric transients is very clear. The same model may be used in asymmetrical stator connections also, as long as the time functions of V_d and V_q can be obtained. But, $v_d(t)$ and $v_q(t)$ may be defined only if $V_A(t)$, $V_B(t)$, $V_C(t)$ functions are available. Alternatively, the load–voltage–current relationships have to be amenable to the state-space form. Let us illustrate this idea with a few examples.

5.17.1 Line-to-Line Fault

A typical line fault at machine terminals is shown in Figure 5.22.

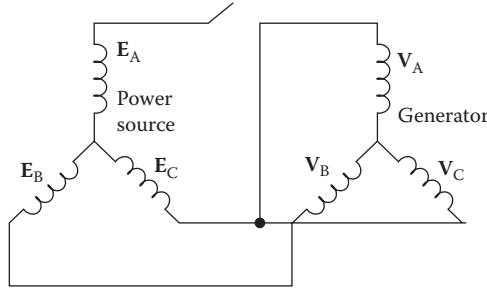


FIGURE 5.22 Line-to-line synchronous generator short-circuit.

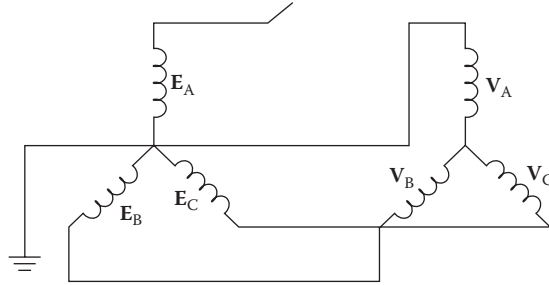


FIGURE 5.23 Line-to-neutral fault.

The power source-generator voltage relationships after the short-circuit are as follows:

$$\begin{aligned} V_B - V_C &= E_B - E_C \\ V_A &= V_C; I_A + I_B + I_C = 0, V_0 = 0 \end{aligned} \quad (5.196)$$

Consequently,

$$\begin{aligned} V_C(t) &= \frac{1}{3}(E_C - E_B) \\ V_B(t) &= -2V_C(t) \\ V_A(t) &= V_C(t) \\ E_{A,B,C}(t) &= V\sqrt{2} \cos\left(\omega_{lt} - (i-1)\frac{2\pi}{3}\right); i=1,2,3 \end{aligned}$$

All stator voltages $V_A(t)$, $V_B(t)$, and $V_C(t)$ may be defined right after the short-circuit.

5.17.2 Line-to-Neutral Fault

In this case, a phase of the synchronous machine is connected to the neutral power system (Figure 5.23), which may or may not be connected to the ground. According to Figure 5.23,

$$\begin{aligned} V_B - V_C &= E_B - E_C \\ V_C - V_A &= E_C; V_A + V_B + V_C = 0 \end{aligned} \quad (5.197)$$

Consequently,

$$\begin{aligned} V_A &= -\frac{(E_C + E_B)}{3} \\ V_B &= V_A + E_B \\ V_C &= V_A + E_C \end{aligned} \tag{5.198}$$

So, again, provided that E_A , E_B , and E_C are known, time functions $V_A(t)$, $V_B(t)$, and $V_C(t)$ are also known.

5.18 Finite Element SG Modeling

The numerical methods for field distribution calculation in electric machines are by now an established field with a rather long history, even before 1975 when finite difference methods prevailed. Since then, the finite element (integral) methods (FEM) took over to produce revolutionary results.

For the basics of the FEM, see the literature [19]. In 1976, in Reference [20], SG time domain responses at standstill were approached successfully by FEM, making use of the conductivity matrices concept. In 1980, the SG sudden three-phase short-circuit was calculated [21,22] by FEM.

The relative motion between stator and rotor during balanced and unbalanced short-circuit transients was reported in 1987 [23].

In the 1990s, the time stepping and coupled-field and circuit concepts were successfully introduced [24] to eliminate circuit simulation restrictions based in conductivity matrices representations. Typical results related to no-load and steady-state short-circuit curves obtained through FEM for a 150 MVA 13.8 kV SG are shown in Figure 5.24, modeled after Reference [4].

Also for steady state, FEMs were proved to predict correctly (within 1 to 2%) the field current required for various active and reactive power loads over the whole $p-q$ capability curve of the same SG [4].

Finally the rotor angle during steady state was predicted within 2° for the whole spectrum of active and reactive power loads (Figure 5.25) [4].

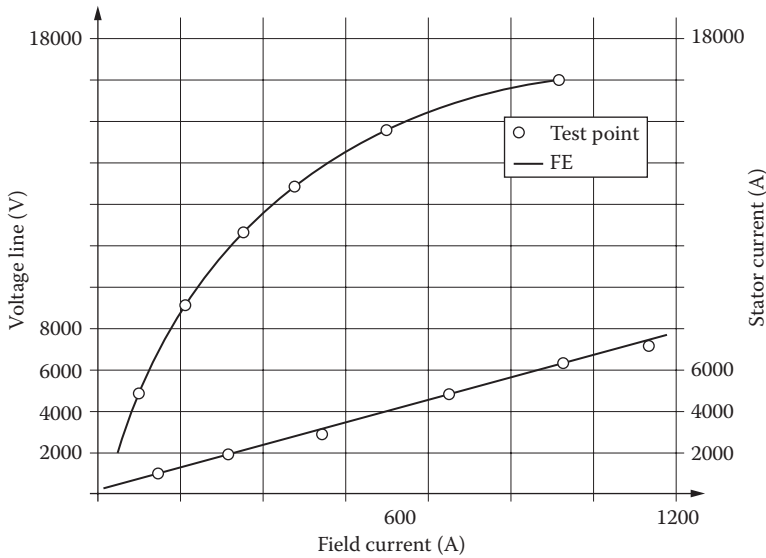


FIGURE 5.24 Open and short-circuit curves of a 150 MVA, 13.8 kV, two-pole synchronous generator.

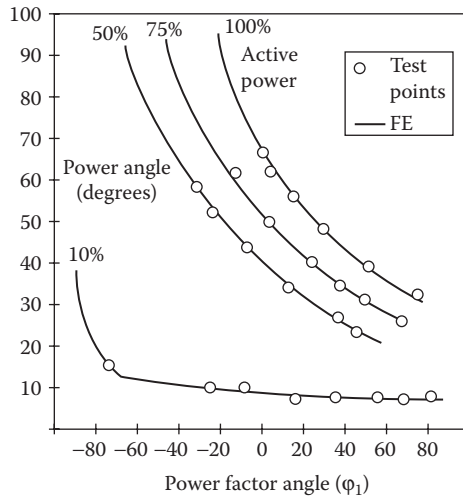


FIGURE 5.25 Power angle δ_v vs. power factor angle ϕ_1 of a 150 MVA, 13.8 kV, two-pole synchronous generator.

The FEM has also been successful in calculating SG response to standstill time domain and frequency responses, then used to identify the general equivalent circuit elements [25,26].

A complete picture of finite element (FE) flux paths during an ongoing sudden short-circuit process is shown in Figure 5.26 [23]. The traveling field is visible. Also visible is the fact that quite a lot of flux paths cross the slots, as expected.

Finally, FE simulation of a 120 MW, 13.8 kV, 50 Hz SG on load during and after a three-phase fault was successfully conducted [28] (Figure 5.27a through Figure 5.27e).

This is a severe transient, as the power angle reaches over 90° during the transients when notable active power is delivered, as the field current is also large. The plateau in the line voltage recovery from 0.4 to 0.5 sec (Figure 5.27d, Figure 5.27e and Figure 5.27c) is explainable in this way.

It is almost evident that FEM has reached the point of being able to simulate virtually any operation mode in an SG. The only question is the amount of computation time required, not to mention the extraordinary expertise involved in wisely using it. The FEM is the way of the future in SG, but there may be two avenues to that future:

- Use FEM to calculate and store SG lumped parameters for an ever-wider saturation and frequency effect and then use the general equivalent circuits for the study of various transients with the machine alone or incorporated in a power system.
- Use FEM directly to calculate the electromagnetic and mechanical transients through coupled-field circuit models or even through powers, torques, and motion equations.

While the first avenue looks more practical for power system studies, the second may be used for the design refinements of a new SG.

The few existing dedicated FEM software packages are expected to improve further, at reduced additional computation time and costs.

5.19 SG Transient Modeling for Control Design

In Section 5.10, simplified models for power system studies were described. One specific approximation has gained wide acceptance, especially for SG control design. It is related to the neglecting of stator transients, neglecting the damper winding effects altogether (third-order model), or considering one damper winding along axis q but no damper winding along axis d (fourth-order model).

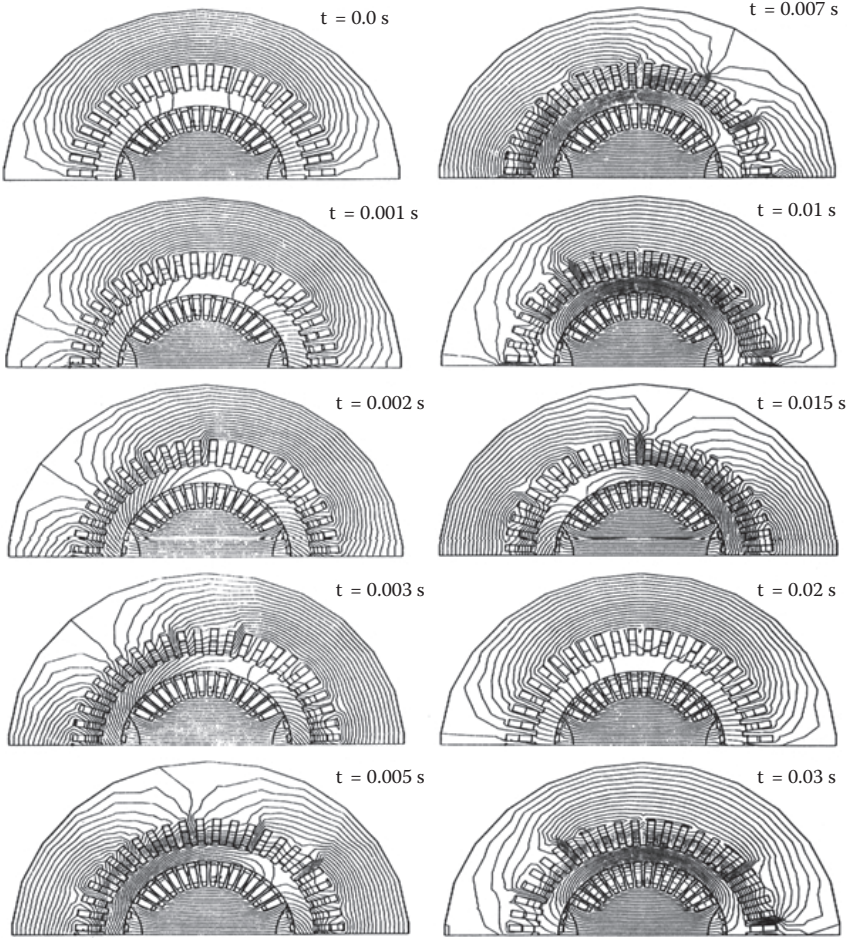


FIGURE 5.26 Flux distribution during a 0.5 per unit (P.U.) balanced short-circuit at a 660 MW synchronous generator terminal (contour intervals are 0.016 Wb/m).

We start with Equation 5.161, the third-order model:

$$V_d = -I_d r_s + \Psi_q \omega_r$$

$$V_q = -I_q r_s - \Psi_d \omega_r$$

$$\begin{bmatrix} \Psi_d \\ \Psi_f \end{bmatrix} = \begin{bmatrix} l_d & l_{dm} \\ l_{dm} & l_f \end{bmatrix} \begin{bmatrix} I_d \\ I_f \end{bmatrix}; \Psi_q = l_q I_q$$

$$\frac{1}{\omega_b} \dot{\Psi}_f = -I_f r_f + v_f \quad (5.199)$$

$$t_e = -(\Psi_d I_q - \Psi_q I_d)$$

$$2H \dot{\omega}_r = t_{shaft} - t_e;$$

$$\omega_b \dot{\delta} = \omega_r - \omega_1$$

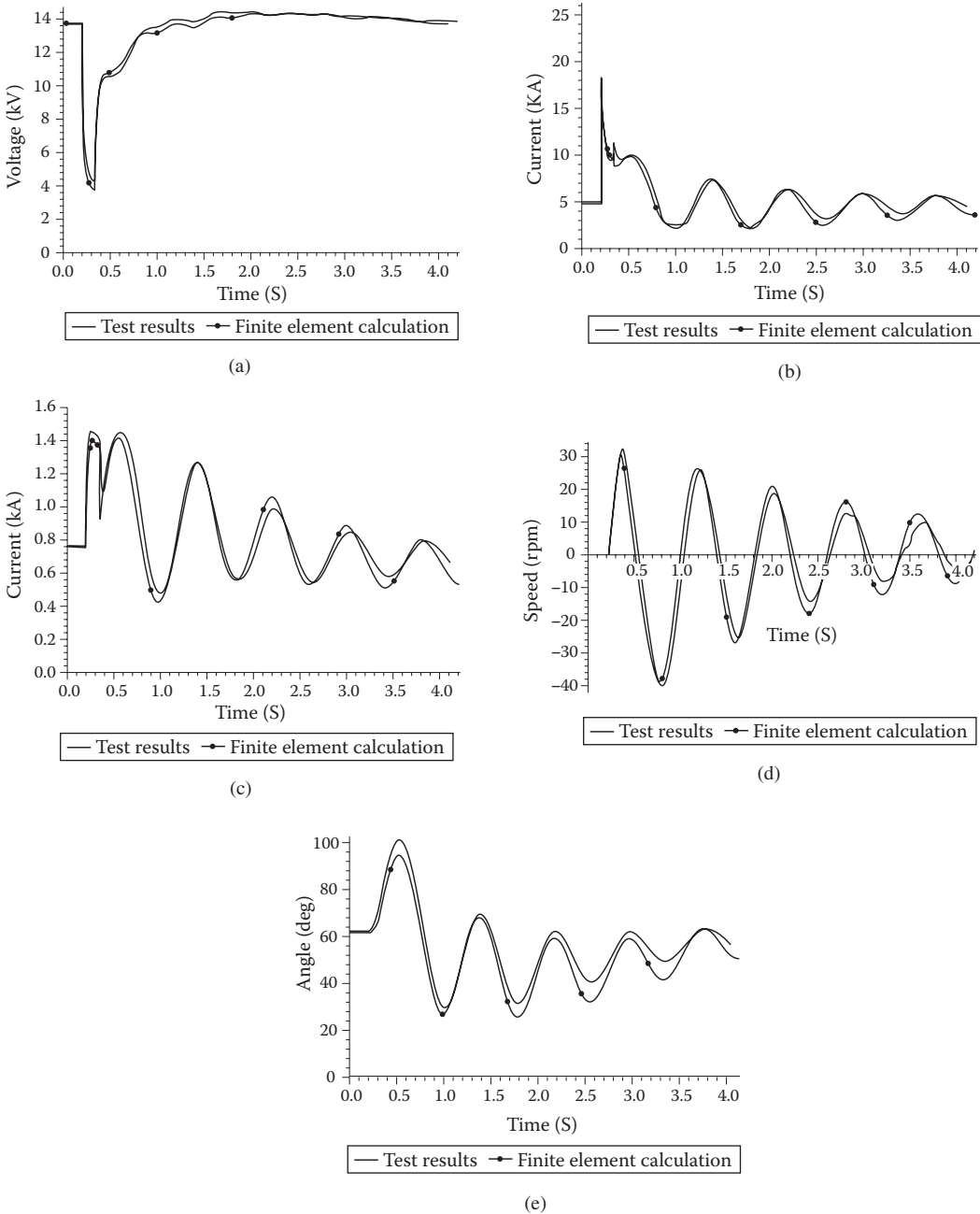


FIGURE 5.27 On-load three-phase fault and recovery transients by finite element and tests: (a) line voltage, (b) line current, (c) field current, (d) rotor speed deviation, and (e) power angle.

Eliminate I_f from the flux–current relationships, and derive the transient emf e'_q as follows:

$$e'_q = +\omega_r \frac{l_{dm}}{l_f} \Psi_f \tag{5.200}$$

the q – axis stator equation becomes

$$V_q = -r_s I_q - x'_d I_d - e'_q \quad (5.201)$$

where

$$x'_d = \omega_r \left(l_d - \frac{l_{dm}^2}{l_f} \right); \quad l_f = l_{\mu} + l_{dm} \quad (5.202)$$

We may now rearrange the field circuit equation in Equation 5.199 with e'_q and x'_d to obtain the following:

$$\dot{e}'_q = \frac{v_f - e'_q + i_d(x_d - x'_d)}{T'_{d0}}; \quad T'_{d0} = \frac{l_f}{r_f} \quad (5.203)$$

By considering a damper winding Q along axis q , an equation similar to Equation 5.200 is obtained, with e'_d as follows:

$$e'_d = \omega_r \frac{l_{qm}}{l_Q} \Psi_Q; \quad (5.204)$$

$$l_Q = l_{Ql} + l_{qm}$$

$$\dot{e}'_d = \frac{-i_q(x_q - x'_q) - e'_d}{T'_{q0}}; \quad T'_{q0} = \frac{l_Q}{r_Q}; \quad x'_q = l_q - \frac{l_{qm}^2}{l_Q} \quad (5.205)$$

$$V_d = -r_s I_d - x'_q I_q - e'_d$$

As expected, in the absence of the rotor q axis damping winding, e'_d is taken as zero, and the third order of the model is restored. x'_d and x'_q are the transient reactances (in P.U.) as defined earlier in this chapter. The two equations of motion in Equation 5.199 and Equation 5.203 and Equation 5.204 have to be added to form the fourth order (transient model) of the SG.

As the transient emf e'_q differential equation includes the field-winding voltage v_f , the application of this model to control is greatly facilitated, as shown in [Chapter 6](#), which is dedicated to the control of SG.

The initial values of transient emfs e'_d and e'_q are calculated from stator equations in Equation 5.201 and Equation 5.204 for steady state:

$$\begin{aligned} (e'_q)_{t=0} &= \omega_{r0} \left(\frac{l_{dm}}{l_f} \cdot I_f(I_f)_{t=0} + l_{dm}(I_d)_{t=0} \right) \\ (e'_d)_{t=0} &= \omega_{r0} \frac{l_{qm}}{l_Q} \cdot (I_q)_{t=0} \end{aligned} \quad (5.206)$$

In a similar way, a subtransient model may be defined for the first moments after a fast transient process [2]. Such a model is not suitable for control design purposes, where the excitation current control is rather slow anyway. Linearization of the transient model with the rotor speed ω_r as variable (even in the stator equations) proves to be very useful in automatic voltage regulator (AVR) design, as shown in Chapter 6.

5.20 Summary

- SGs undergo transient operation modes when the currents, flux linkages, voltages, load, or speed vary with time. Connection of the SGs to a power system can result in electrical load or shaft torque variations that produce transients. The steady-state, two-reaction, model in [Chapter 4](#), based on traveling fields at standstill to each other, is valid only for steady-state operation.
- The main SG models for transients are as follows:
 - The phase-variable circuit model is based on the SG structure, as multiple electric and magnetic circuits are coupled together electrically and/or magnetically. The stator/rotor circuit mutual inductances always depend on rotor position. In salient-pole rotors, the stator phase self-inductances vary with rotor position, that is, with time. With one damper circuit along each rotor axis, a field winding, and three stator phases, an eighth-order nonlinear system with time-variable coefficients is obtained, when the two equations of motion are added. The basic variables are $I_A, I_B, I_C, I_f, I_D, I_Q, \omega_r, \theta_r$.
 - Solving a variable coefficient state-space system requires inversion of a time-variable sixth-order matrix for each integration step time interval. Only numerical methods such as Runge–Kutta–Gill and predictor–corrector can handle the solving of such a stiff state-space high-order system. In addition, the computation time is prohibitive due to the time dependence of inductances. Finally, the complexity of the model leaves little room for intuitive interpretation of phenomena and trends for various transients. And, it is not practical for control design. In conclusion, the phase-variable model should be used only for special situations, such as for highly unbalanced transients or faults.
 - Simpler models are needed to handle transients in a more practical manner.
- The orthogonal axis (d – q) model is characterized by inductances between windings which are independent from rotor position. The d – q model may be derived from the phase-variable model either through a mathematical change of variables (Park transform) or through a physical orthogonal axis model.
 - The Park transform is an orthogonal change of variables such that to conserve powers, losses, and torque,

$$\begin{aligned}
 \begin{bmatrix} V_d \\ V_q \\ V_0 \end{bmatrix} &= \begin{bmatrix} P(\theta_{er}) \end{bmatrix} \cdot \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} \\
 \begin{bmatrix} P(\theta_{er}) \end{bmatrix} &= \frac{2}{3} \begin{bmatrix} \cos(-\theta_{er}) & \cos\left(-\theta_{er} + \frac{2\pi}{3}\right) & \cos\left(-\theta_{er} - \frac{2\pi}{3}\right) \\ \sin(-\theta_{er}) & \sin\left(-\theta_{er} + \frac{2\pi}{3}\right) & \sin\left(-\theta_{er} - \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad (5.207) \\
 \begin{bmatrix} P(\theta_{er}) \end{bmatrix}^{-1} &= \frac{3}{2} \begin{bmatrix} P(\theta_{er}) \end{bmatrix}^T; \frac{d\theta_{er}}{dt} = \omega_r
 \end{aligned}$$

- The physical d – q model consists of a fictitious machine with the same rotor f, D, Q orthogonal rotor windings as in the actual machine and with two stator windings with magnetic axes (mmfs) that are always fixed to the rotor d and q orthogonal axes. The fact that the rotor d – q axes move at rotor speed and are always aligned with axes d and q secure the independence of the d – q model inductances of rotor position.

- Steady state means DC in the d - q model of SG.
- For complete equivalence of the d - q model with the real machine, a null component is added. This component does not produce torque through the fundamental, and its current is determined by the stator resistance and leakage inductance:

$$V_0 = -r_s I_0 - l_{sl} \frac{s}{\omega_b} \frac{dI_0}{dt} \quad (5.208)$$

- The dependence of the d - q model parameters on the real machine parameters is rather simple.
- To reduce the number of inductances in the d - q model, the rotor quantities are reduced to the stator under the assumption that the airgap main magnetic field couples all windings along axes d and q , respectively. Thus, the stator-rotor coupling inductances become equal to the stator magnetization inductances l_{dm} , l_{qm} .
- An additional leakage coupling inductance l_{pl} between the field winding and the d axis damper winding is also introduced, as it proves to be useful in providing good results both in the stator and field current transient responses.
- The d - q model is generally used in the per unit (P.U.) form to reduce the range of variables during transients from zero to, say, 20 for all transients and all power ranges. The base quantities, voltage, current, and power are rated, in general, but other choices are possible. In this chapter, all variables and parameters are in P.U., but time t and inertia H are in seconds. In this case, $d/dt \rightarrow s/\omega_b$ in Laplace form (ω_b is equal to the base [rated] angular frequency in rad/sec).
- The rotor-to-stator reduction coefficients conserve losses and power, as expected.
- The d - q model equations writing assumes source circuits in the stator and sink circuits in the

rotor and the induced voltage $e = -d\Psi/dt$. Also, implicitly, the Poynting vector $\bar{P} = \frac{\bar{E} \times \bar{H}}{2}$ enters

the sink circuits and exits the source circuits. This way, all flux/current relations contain only positive (+) signs. This choice leads to the fact that while power and torque are positive for generating, the components V_d , V_q , and I_q are always negative for generating. But, I_d can be either negative or positive depending on the load power factor angle. This is valid for the trigonometric positive motion direction.

- The space-vector diagram at steady-state evidences the power angle δ_v between the voltage vector and axis q in the third quadrant, with $\delta_v > 0$ for generating.
- Based on the d - q model, state-space equations in P.U. (Equation 5.81), two distinct general equivalent circuits may be drawn (Figure 5.6). They are very intuitive in the sense that all d - q model equations may be derived by inspection. The distinct d and q equivalent circuits for transients indicate that there is no magnetic coupling between the two orthogonal axes.
- In reality, in heavily saturated SGs, there is a cross-coupling due to magnetic saturation between the two orthogonal axes. Putting this phenomenon into the d - q model has received a lot of attention lately, but here, only two representative solutions are described.
- One uses distinct but unique magnetization curves along axes d and q : $\Psi_{dm}^*(I_m)$, $\Psi_{qm}^*(I_m)$, where I_m is the total magnetization current (or mmf): $I_m = \sqrt{I_{dm}^2 + I_{qm}^2}$. This approximation seems to fail when the SG is “deep” in the leading power factor mode (underexcited, with $I_m < 0.7$ P.U.). However, when $I_m > 0.7$ P.U., it has not yet been proven wrong.
- The second model for saturation presupposes a family of magnetization curves along axis d and, respectively, q : $\Psi_{dm}^*(I_{dm}, I_{qm})$, $\Psi_{qm}^*(I_{dm}, I_{qm})$. This model, after adequate analytical approximations of these functions, should not fail over the entire active/reactive power envelope of an SG. But, it requires more computation efforts.
- The magnetization curves along axes d and q may be obtained either from experiments or through FEM field calculations.

- The cross-coupling magnetic saturation may be handily included in the d - q general equivalent circuit for both axes (Figure 5.21).
- The general equivalent circuits are based on the flux/stator current relationships without rotor currents:

$$\begin{aligned}\Psi_d(s) &= l_d(s)I_d(s) + g(s)v_f(s) \\ \Psi_q(s) &= l_q(s)I_q(s)\end{aligned}\quad (5.209)$$

- $l_d(s)$, $l_q(s)$ and $g(s)$ are known as the operational parameters of the SG (s equals the Laplace operator).
- Operational parameters include the main inductances and time constants that are catalog data of the SG: subtransient, transient, and synchronous inductances l_d'' , l_d' , l_d , along the d axis and l_q'' , l_q' along the q axis. The corresponding time constants are T_d'' , T_d' , T_{d0}'' , T_{d0}' , T_q'' , T_{q0}' . Additional terms are added when frequency (skin) effect imposes additional fictitious rotor circuits: l_d''' , l_q''' , l_q' , T_d''' , T_{q0}''' , T_q''' , T_{q0}''' .
- Though transients may be handled directly via the complete d - q model through its solving by numerical methods, a few approximations have led to very practical solutions.
- Electromagnetic transients are those that occur at constant speed. The operational calculus may be applied with some elegant analytical approximated solutions as a result. Sudden three-phase short-circuit falls into this category. It is used for unsaturated parameter identification by comparing the measured and calculated current waveforms during a sudden short-circuit. Graphical models (20 years ago) and advanced nonlinear programming methods have been used for curve fitting the theory and tests, with parameter estimation as the main goal.
- Electromagnetic transients may also be provoked at zero speed with the applied voltage vector along axis d or q with or without DC in the other axis. The applied voltage may be DC step voltage or PWM cyclical voltage rich in frequency content. Alternatively, single-frequency voltages may be applied one after the other and the impedance measured (amplitude and phase). Again, a way to estimate the general equivalent circuit parameters was born through standstill electromagnetic transient processing. Alternatively, FEM calculations may replace or accompany tests for the same purpose: parameter estimation.
- For multimachine transient investigation, simpler lower-order d - q models have become standard. One of them is based on neglecting the stator (fast) transients. In this model, the fast-decaying AC components in stator currents and torque are missing. Further, it seems better that in this case, the rotor speed in the stator equations be kept equal to the synchronous speed $\omega_r = \omega_{r0} = \omega_1$. The speed in the model varies as the equation of motion remains in place.
- Gradually, the damper circuit transients may also be neglected ($I_D = I_Q = 0$); thus, only one electrical transient, determined by the field winding, remains. With the two motion equations, this one makes for a third-order system. Finally, all electric transients may be neglected to be left only with the motion equations, for very slow (mechanical) transients.
- Asynchronous running at constant speed may also be tackled as an electromagnetic transient, with $S \rightarrow js\omega_1$; S equals slip; and $S = (\omega_1 - \omega_r)/\omega_1$. An inflexion in the asynchronous torque is detected around $S = 1/2$ ($\omega_r = \omega_1/2$). It tends to be small in large machines, as r_s is very small in P.U.
- In close to synchronous speed, $|S| < 0.05$, the asynchronous torque is proportional to slip speed $S\omega_1$. Also, torque pulsations occur in the asynchronous torque that have to be accounted for during transients, for better precision, as asynchronous torque t_{as} is

$$t_{as} = t_{asav} + t_{asp} \cos(2S\omega_1\omega_b + \Psi) \quad (5.210)$$

- Its pulsation frequency is small, because S gets smaller, as it is the case around synchronism ($|S| < 0.05$, $\omega_r \approx (0.95 - 1.05)\omega_1$).

- Mechanical (very small) transients may be treated easily with the SG at steady state electromagnetically. Only speed ω_r and power angle δ_v vary in the so-called rotor swing equation. Through numerical methods, the nonlinear model may be solved, but for small perturbations, a simple analytical solution for $\omega_r(t)$ and $\delta_v(t)$ may be found for particular inputs, such as shaft torque step or frequency response.
- For the SG in stand-alone operation without a damper cage, a proper mechanical frequency f_0 is defined. It varies with field current I_f , power angle δ_v , and inertia, but it is generally less than 2 to 3 Hz for large and medium power standard SGs. When an SG is connected to the power grid and the shaft torque presents pulsations at Ω_v , resonance conditions may occur. They are characterized by severe oscillation amplifications (in δ_v) that are tempered by a large inertia and a strong damper cage. Torsional shaft frequencies by the prime-mover shaft and coupling to the SG may also produce resonance mechanical conditions that have to be avoided.
- Electromechanical transients are characterized by both electrical and mechanical transients. For small perturbations, the d - q model provides a very good way to investigate the SG stability — without or with voltage and speed control — by the eigenvalue method, after linearization around a steady-state given point.
- For large disturbance transients, the full d - q model with magnetic saturation and frequency effects (Figure 5.21) is recommended. Numerical methods may solve the transients, but direct stability methods typical to nonlinear systems may also be used.
- Finite element analysis is widely used to assess various SG steady state and transients through coupled field/circuit models. The computation time is still prohibitive for use in design optimization or for controller design. With the computation power of microcomputers rising by the year, the FEM will become the norm in analyzing the FEM steady-state and transient performance: electromagnetic, thermal, or mechanical.
- Still, the circuit models, with the parameters calculated through FEM and then curve fitted by analytical approximations, will eventually remain the norm for preliminary and optimization design, particular transients, and SG control.
- The approximate (circuit) transient (fourth-order) model of SG is finally given, as it will be used in Chapter 6, which is dedicated to the control of SGs.

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